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Mathematik positiv!

Neuausgabe für die **Zentralmatura 2014**

6. Klasse AHS

Lösungen

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A. POTENZEN – WURZELN

1 (a) $3^4 = 3\cdot 3\cdot 3\cdot 3 = 81$ (b) $(-5)\cdot(-5)\cdot(-5) = -125$ (c) $(-3)\cdot(-3)\cdot(-3)\cdot(-3) = 81$

(d) $0{,}3\cdot 0{,}3\cdot 0{,}3 = 0{,}027$ (e) $(-1{,}2)\cdot(-1{,}2) = 1{,}44$ (f) $3{,}2\cdot 3{,}2\cdot 3{,}2 = 32{,}768$

(g) $\left(-\frac{1}{2}\right)\cdot\left(-\frac{1}{2}\right)\cdot\left(-\frac{1}{2}\right)\cdot\left(-\frac{1}{2}\right) = \frac{1}{16}$ (h) $\left(-\frac{7}{5}\right)\cdot\left(-\frac{7}{5}\right) = \frac{49}{25}$ (i) $\left(\frac{5}{2}\right)\cdot\left(\frac{5}{2}\right)\cdot\left(\frac{5}{2}\right) = \frac{125}{8}$

(j) $\frac{8}{9}$ (k) $\frac{-64}{5}$ (l) $\left(-\frac{2}{5}\right)\cdot\left(-\frac{2}{5}\right)\cdot\left(-\frac{2}{5}\right) = -\frac{8}{125}$

2 (a) 16 (b) -16 (c) -64 (d) -125 (e) a^2 (f) $-a^5$

3 (a) $a^{3+4} = a^7$ (b) $b^{2+6} = b^8$

(c) $x^{2+1}\cdot y^{2+2} = x^3y^4$ (d) $-u^3\cdot v^6\cdot u^2 = -u^{3+2}\cdot v^6 = -u^5v^6$

(e) $9^{7-5} = 9^2 = 81$ (f) $2^8 : \left(-2^3\right) = -2^{8-3} = -2^5 = -32$

(g) $\left(\frac{1}{3}\right)^4 : \left(\frac{1}{3}\right)^3 = \left(\frac{1}{3}\right)^{4-3} = \left(\frac{1}{3}\right)^1 = \frac{1}{3}$ (h) $-5^5 : 5^2 = -5^{5-2} = -5^3 = -125$

4 (a) $a^{n+1+2n+3} = a^{3n+4}$ (b) $b^{n-1}\cdot b^2\cdot b^{2n-1} = b^{n-1+2+2n-1} = b^{3n}$

(c) $x^{2n-1}\cdot y^{2n}\cdot x^{2n+1}\cdot y = x^{4n}y^{2n+1}$ (d) $u^{2+n}v^{n+1}\cdot u^{1-n}v^{2n+3} = u^3v^{3n+4}$

(e) $a^7 : a^5 = a^2$ (f) $(-x)^9 : x^4 = (-1)\cdot x^{9-4} = -x^5$

(g) $\left(-\frac{1}{b}\right)^4 : \left(\frac{1}{b}\right)^2 = (-1)^4\cdot\left(\frac{1}{b}\right)^4 : \left(\frac{1}{b}\right)^2 = \left(\frac{1}{b}\right)^{4-2} = \left(\frac{1}{b}\right)^2 = \frac{1}{b^2}$

(h) $u^{2n+1} : u^{2n+1} = u^{2n+1-(2n+1)} = u^0 = 1$ (i) $a^3 : a^7 = \frac{a^3}{a^7} = \frac{1}{a^{7-3}} = \frac{1}{a^4}$

5 (a) $(a+b)^{2+n}\cdot(a+b)^{3n+1} = (a+b)^{2+n+3n+1} = (a+b)^{4n+3}$

(b) $(x-y)^{2a}\cdot(x-y)^{b-3}\cdot(x-y)^{a-b} = (x-y)^{2a+b-3+a-b} = (x-y)^{3a-3}$

6 (a) $\frac{15a^4b^2}{-3a^3b}$ Kürzen durch $3, a^3, b$: $-5ab$

(b) $\frac{-128x^7y^5z^3}{64x^2y^3z^5}$ Kürzen durch $64, x^2, y^3, z^3$: $\frac{-2x^5y^2}{z^2}$

(c) $\frac{36\cdot(a-b)^4\cdot(x+y)}{24\cdot(a-b)^3}$ Kürzen durch $12, (a-b)^3$: $\frac{3(a-b)(x+y)}{2}$

(d) $\frac{48\cdot(x-y)^4\cdot(x+y)^2}{120\cdot\left(x^2-y^2\right)}$ Kürzen durch $24, (x-y), (x+y)$: $\frac{2(x-y)^3(x+y)}{5}$

7 (a) $\left(x^4\right)^4 = x^{16}$ (b) $\left(-y^3\right)^2 = (-1)^2\left(y^3\right)^2 = y^6$ (c) $\left(3x^3\right)^2 = 9x^6$

(d) $\left[\left(\frac{x}{2}\right)^2\right]^5 = \left(\frac{x}{2}\right)^{10} = \frac{x^{10}}{1024}$ (e) $\left(\frac{2}{5}a^2\right)^3 = \frac{2^3}{5^3}\left(a^2\right)^3 = \frac{8}{125}a^6$ (f) $\left(\frac{-x^2y}{3z^3}\right)^3 = -\frac{x^6y^3}{27z^9}$

8 (a) $\left[\frac{3^2}{2^3}\cdot\left(-\frac{2}{3}\right)^4 + \frac{7}{(-3)^2}\right]\cdot(-3)^2 = \left[\frac{3^2}{2^3}\cdot\frac{2^4}{3^4} + \frac{7}{9}\right]\cdot 9 =$ Kürzen durch $2^3, 3^2$

$= \left[\frac{2}{3^2} + \frac{7}{9}\right]\cdot 9 = \left[\frac{2}{9} + \frac{7}{9}\right]\cdot 9 = \frac{9}{9}\cdot 9 = 9$

(b) $\left[\left(-\frac{3}{2}\right)^3\cdot\left(\frac{4}{5}\right)^2\right]^2 : \frac{9}{5} = \left[-\frac{3^3}{2^3}\cdot\frac{4^2}{5^2}\right]^2\cdot\frac{5}{9} = \left[-\frac{3^3}{2^3}\cdot\frac{2^4}{5^2}\right]^2\cdot\frac{5}{9} =$ Kürzen durch 2^3

$= \left[-\frac{3^3\cdot 2}{5^2}\right]^2\cdot\frac{5}{3^2} = \frac{3^6\cdot 2^2}{5^4}\cdot\frac{5}{3^2} = \frac{3^4\cdot 2^2}{5^3} = \frac{81\cdot 4}{125} = \frac{324}{125}$

(c) $\frac{\left(-a^2b\right)^3}{x^2y}\cdot\left[\frac{x^2y^3}{ab} : \frac{y}{b^2}\right]^2 = \frac{-a^6b^3}{x^2y}\cdot\left[\frac{x^2y^3}{ab}\cdot\frac{b^2}{y}\right]^2 = \frac{-a^6b^3}{x^2y}\cdot\left[\frac{x^2y^2}{a}\cdot\frac{b}{1}\right]^2 =$

$= \frac{-a^6b^3}{x^2y}\cdot\frac{x^4y^4\cdot b^2}{a^2} = -a^4\cdot b^5\cdot x^2\cdot y^3$

(d) $\left(\frac{a-b}{x+y}\right)^3 : \left(\frac{a-b}{x^2-y^2}\right)^2 = \frac{(a-b)^3}{(x+y)^3} : \frac{(a-b)^2}{\left(x^2-y^2\right)^2} = \frac{(a-b)^3}{(x+y)^3} : \frac{(a-b)^2}{\left[(x-y)(x+y)\right]^2} =$

$= \frac{(a-b)^3}{(x+y)^3}\cdot\frac{(x-y)^2(x+y)^2}{(a-b)^2} = \frac{(a-b)(x-y)^2}{(x+y)}$

9 (a) $\left(12x^3-29x^2y+30xy^2-25y^3\right):\left(3x-5y\right)=4x^2-3xy+5y^2$ $\qquad x, y \in \mathbb{R}$ mit $3x-5y \neq 0$

$$\begin{array}{l}
\underline{12x^3-20x^2y} \\
\quad -9x^2y+30xy^2 \\
\quad \underline{-9x^2y+15xy^2} \\
\qquad +15xy^2-25y^3 \\
\qquad \underline{+15xy^2-25y^3} \\
\qquad\qquad 0 \text{ Rest}
\end{array}$$

(b) $\left(5a^4-7a^3b+5a^3b^2-7a^2b^3+ab^3+b^5\right):\left(5a^3-7a^2b+b^3\right)=a+b^2$ $\qquad a, b \in \mathbb{R}, 5a^3-7a^2b+b^3 \neq 0$

$$\begin{array}{l}
\underline{5a^4-7a^3b \qquad\qquad +ab^3} \\
\quad +5a^3b^2-7a^2b^3 \qquad +b^5 \\
\quad \underline{+5a^3b^2-7a^2b^3 \qquad +b^5} \\
\qquad\qquad 0 \text{ Rest}
\end{array}$$

(c) $\left(37x^3-11x^4+42x+3x^5-18-32x^2\right):\left(9+x^2-3x\right)$ Ordnen nach fallenden Potenzen von x

$\left(3x^5-11x^4+37x^3-32x^2+42x-18\right):\left(x^2-3x+9\right)=3x^3-2x^2+4x-2$ $\qquad x, y \in \mathbb{R}, x^2-3x+9 \neq 0$

$$\begin{array}{l}
\underline{3x^5-9x^4+27x^3} \\
\quad -2x^4+10x^3-32x^2 \\
\quad \underline{-2x^4+6x^3-18x^2} \\
\qquad +4x^3-14x^2+42x \\
\qquad \underline{+4x^3-12x^2+36x} \\
\qquad\quad -2x^2+6x-18 \\
\qquad\quad \underline{-2x^2+6x-18} \\
\qquad\qquad 0 \text{ Rest}
\end{array}$$

(d) $\left(6x^4-7x^3y-2xy^3+2x^2y^2+xy+y^4\right):\left(2x-y\right)=3x^3-2x^2y-y^3$ $\qquad x, y \in \mathbb{R}, 2x-y \neq 0$

$$\begin{array}{l}
\underline{6x^4-3x^3y} \\
\quad -4x^3y-2xy^3+2x^2y^2 \\
\quad \underline{-4x^3y \qquad\quad +2x^2y^2} \\
\qquad -2xy^3 \qquad +xy+y^4 \\
\qquad \underline{-2xy^3 \qquad\quad +y^4} \\
\qquad\qquad xy \quad \text{Rest}
\end{array}$$

10 (a) $25x^2-16y^2=\left(5x-4y\right)\left(5x+4y\right)$

(b) $x^3-\frac{1}{8}y^3=\left(x\right)^3-\left(\frac{1}{2}y\right)^3=\left(x-\frac{1}{2}y\right)\left(x^2+\frac{1}{2}xy+\frac{1}{4}y^2\right)$

(c) $\frac{1}{81}a^4-1=\left(\frac{1}{3}a\right)^4-1^4=\left[\left(\frac{1}{3}a\right)^2-1^2\right]\left[\left(\frac{1}{3}a\right)^2+1^2\right]=\left(\frac{1}{3}a-1\right)\left(\frac{1}{3}a+1\right)\left(\frac{1}{9}a^2+1\right)$

(d) $2x^4+54x=2x\left(x^3+27\right)=2x\left(x+3\right)\left(x^2-3x+9\right)$

(e) $a^{2m}-b^{2m}=\left(a^m\right)^2-\left(b^m\right)^2=\left(a^m-b^m\right)\left(a^m+b^m\right)$

(f) $a^{3m}+b^{3m}=\left(a^m\right)^3+\left(b^m\right)^3=\left(a^m+b^m\right)\left(a^{2m}-a^mb^m+b^{2m}\right)$

11 (a) $150\,000\,000:\left(2\cdot 6\,370\right)=11\;773{,}94..$
Man muss rund 11 744 Erdkugeln aneinanderreihen.

(b) $150\,000\,000:\left(1{,}3914\cdot 10^6\right)=107{,}80..$
Man muss rund 108 Sonnen aneinanderreihen.

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12 Masse der Erde: $m = V \cdot \rho$ $\qquad \rho = 5{,}5\ g/cm^3 = 5{,}5\ kg/dm^3 = 5\,500\ kg/m^3$

$V = \frac{4}{3} r^3 \pi$

$V = \frac{4}{3} \cdot 6\,370^3 \cdot \pi$

$V = 1{,}0826969.. \cdot 10^{12}\ km^3$ $\qquad 1\,km^3 = 10^9\ m^3$

$V = 1{,}0826969.. \cdot 10^{12} \cdot 10^9\ m^3$

$V = 1{,}0826969.. \cdot 10^{21}\ m^3$

$m = 1{,}0826969.. \cdot 10^{21} \cdot 5\,500\ kg$

$m = 5{,}9548.. \cdot 10^{24}\ kg$

$m : 70 = 8{,}5069.. \cdot 10^{22}$

Die Masse der Erde entspricht der Masse von rund $8{,}5069 \cdot 10^{22}$ Menschen.

13 (a) 2 GB = $2 \cdot 10^3$ MB

$2 \cdot 10^3 : 2{,}75 = 727{,}27..$

Auf einer 2 GB-Speicherkarte kann man rund 727 Fotos speichern.

(b) 16 GB = $16 \cdot 10^3$ MB

$16 \cdot 10^3 : 2{,}75 = 5\,818{,}18..$

Auf einer 16 GB-Speicherkarte kann man rund 5 818 Fotos speichern.

14 (a) $1^{-1} = 1$ (b) $2^{-6} = \frac{1}{2^6} = \frac{1}{64}$ (c) $6^{-2} = \frac{1}{6^2} = \frac{1}{36}$ (d) $4^{-3} = \frac{1}{4^3} = \frac{1}{64}$

(e) $9 \cdot 3^{-2} = 9 \cdot \frac{1}{3^2} = 9 \cdot \frac{1}{9} = 1$ (f) $16 \cdot 4^{-3} = 16 \cdot \frac{1}{4^3} = 16 \cdot \frac{1}{64} = \frac{1}{4}$

(g) $25^3 \cdot 5^{-4} = 5^6 \cdot 5^{-4} = 5^2 = 25$ (h) $0{,}4^{-1} = \left(\frac{4}{10}\right)^{-1} = \frac{10}{4} = \frac{5}{2}$

15 (a) $1^0 = 1$ (b) $a^4 \cdot a^0 = a^4 \cdot 1 = a^4$

(c) $3a^0 = 3 \cdot 1 = 3$ (d) $(3a)^0 = 1$

(e) $3 \cdot (a-b)^0 = 3 \cdot 1 = 3$ (f) $\left(a^{2x-1}\right)^0 = 1$

(g) $(x-3) \cdot x^0 = (x-3) \cdot 1 = x-3$ (h) $4 \cdot \left(5a^0 + a\right)^0 = 4 \cdot 1 = 4$

16 (a) $\left(\frac{1}{4}\right)^{-2} = 4^2 = 16$ (b) $\frac{1}{3^{-4}} = 3^4 = 81$

(c) $\left(\frac{1}{2}\right)^{-5} = 2^5 = 32$ (d) $\left(\frac{5}{8}\right)^{-3} = \left(\frac{8}{5}\right)^3 = \frac{512}{125}$

(e) $\left(\frac{15}{16}\right)^{-1} = \frac{16}{15}$ (f) $\left(\frac{2}{3}\right)^{-2} = \frac{9}{4}$

(g) $0{,}2^{-1} = \frac{10}{2} = 5$ (h) $0{,}4^{-3} = \left(\frac{4}{10}\right)^{-3} = \left(\frac{10}{4}\right)^3 = \left(\frac{5}{2}\right)^3 = \frac{125}{8}$

(i) $0{,}8^{-4} = \left(\frac{8}{10}\right)^{-4} = \left(\frac{10}{8}\right)^4 = \left(\frac{5}{4}\right)^4 = \frac{625}{256}$ (j) $(-0{,}16)^{-2} = \left(-\frac{16}{100}\right)^{-2} = \left(-\frac{100}{16}\right)^2 = \left(-\frac{25}{4}\right)^2 = \frac{625}{16}$

(k) $0{,}04^{-2} = \left(\frac{4}{100}\right)^{-2} = \left(\frac{100}{4}\right)^2 = 25^2 = 625$ (l) $(-0{,}02)^{-3} = \left(-\frac{2}{100}\right)^{-3} = \left(-\frac{100}{2}\right)^3 = (-50)^3 = -125\,000$

17 (a) $y^{-2} = \frac{1}{y^2}$ (b) $z^{-3} = \frac{1}{z^3}$ (c) $4x^{-5} = \frac{4}{x^5}$ (d) $7b^{-4} = \frac{7}{b^4}$

(e) $x^3 \cdot y^{-4} = \frac{x^3}{y^4}$ (f) $x^{-2} \cdot y^3 = \frac{y^3}{x^2}$ (g) $2x^4 \cdot z^{-3} = \frac{2x^4}{z^3}$ (h) $x^{-5} \cdot y^{-2} \cdot z^8 = \frac{z^8}{x^5 y^2}$

(i) $2^{-4} \cdot x^2 \cdot y^{-3} \cdot z^4 = \frac{x^2 z^4}{16y^3}$ (j) $\frac{3}{x^{-4}} = 3x^4$

(k) $\frac{5^{-1}}{u^{-2}} = \frac{u^2}{5}$ (l) $\frac{3^{-2}}{t^{-3}} = \frac{t^3}{9}$

18 (a) $4 \cdot x^{-2} = \frac{4}{x^2}$ (b) $a^{-3} \cdot b \cdot c^2 = \frac{bc^2}{a^3}$ (c) $(2 \cdot x)^{-3} = \frac{1}{8x^3}$ (d) $x^{-3} \cdot y \cdot z^{-2} = \frac{y}{x^3 z^2}$

(e) $\frac{3 \cdot x^2}{y \cdot z^{-3}} = \frac{3x^2 z^3}{y}$ (f) $\frac{5 \cdot x^{-3}}{y^{-2} \cdot z} = \frac{5y^2}{x^3 z}$ (g) $\frac{3 \cdot a \cdot b^{-3}}{c^{-2} \cdot d^2} = \frac{3ac^2}{b^3 d^2}$ (h) $\frac{2 \cdot x^{-2} \cdot y^3 \cdot z^{-4}}{3 \cdot x^2 \cdot z} = \frac{2y^3}{3x^4 z^5}$

19 (a) $3^2 \cdot 3^{-4} = 3^{-2} = \frac{1}{3^2} = \frac{1}{9}$

(b) $5^{-3} \cdot 5^2 \cdot 5^4 = 5^3 = 125$

(c) $5^{-3} \cdot 25^{-2} \cdot 5^5 = 5^{-3} \cdot \left(5^2\right)^{-2} \cdot 5^5 =$

$= 5^{-3} \cdot 5^{-4} \cdot 5^5 = 5^{-2} = \frac{1}{5^2} = \frac{1}{25}$

(d) $3^{-2} \cdot 9^2 \cdot 27^{-2} = 3^{-2} \cdot \left(3^2\right)^2 \cdot \left(3^3\right)^{-2} =$

$= 3^{-2} \cdot 3^4 \cdot 3^{-6} = 3^{-4} = \frac{1}{3^4} = \frac{1}{81}$

(e) $2^4 \cdot 2^{-3} \cdot 2^0 = 2$

(f) $\left(\frac{1}{3}\right)^{-2} \cdot \left(\frac{1}{3}\right)^3 \cdot \left(\frac{1}{3}\right)^{-1} = \left(\frac{1}{3}\right)^0 = 1$

20 (a) $x^{-3} \cdot x^5 = x^2$

(b) $x^2 \cdot x^4 \cdot x^{-3} = x^3$

(c) $\frac{a^{-2}}{a^{-5}} = a^{-2-(-5)} = a^{-2+5} = a^3$

(d) $\frac{a^{-n}}{a^{2n}} = a^{-n-2n} = a^{-3n} = \frac{1}{a^{3n}}$

(e) $\frac{b^n}{b^{-3n} \cdot a^n} = \frac{b^n \cdot b^{3n}}{a^n} = \frac{b^{n+3n}}{a^n} = \frac{b^{4n}}{a^n}$

(f) $\frac{(x-y)^n}{(x-y)^{-3n}} = (x-y)^{n-(-3n)} = (x-y)^{n+3n} = (x-y)^{4n}$

21 (a) $\frac{2 \cdot 3^{-2} \cdot 5^{-2}}{3^2 \cdot 2^{-2} \cdot 15^{-2}} = \frac{2 \cdot 2^2 \cdot 15^2}{3^2 \cdot 3^2 \cdot 5^2} = \frac{2^3 \cdot (3 \cdot 5)^2}{3^4 \cdot 5^2} = \frac{2^3 \cdot 3^2 \cdot 5^2}{3^4 \cdot 5^2} = \frac{2^3}{3^2} = \frac{8}{9}$

(b) $\frac{4^{-2} \cdot 12^2 \cdot 15^2 \cdot 7^{-2}}{3^2 \cdot 5^{-2} \cdot 125} = \frac{12^2 \cdot 15^2 \cdot 5^2}{4^2 \cdot 7^2 \cdot 3^2 \cdot 125} = \frac{(3 \cdot 4)^2 \cdot (3 \cdot 5)^2 \cdot 5^2}{4^2 \cdot 7^2 \cdot 3^2 \cdot 5^3} = \frac{\cancel{3^2} \cdot \cancel{4^2} \cdot 3^2 \cdot \cancel{5^2} \cdot \cancel{5^2}^{\,1}}{\cancel{4^2} \cdot 7^2 \cdot \cancel{3^2} \cdot \cancel{5^3}} = \frac{3^2 \cdot 5}{7^2} = \frac{45}{49}$

(c) $\frac{(-2)^2 \cdot (-6)^{-3} \cdot 3^2 \cdot 9^{-3}}{5^2 \cdot 10^{-2} \cdot 12^{-3}} = \frac{(-2)^2 \cdot 3^2 \cdot 10^2 \cdot 12^3}{(-6)^3 \cdot 9^3 \cdot 5^2} = \frac{(-2)^2 \cdot 3^2 \cdot (2 \cdot 5)^2 \cdot (3 \cdot 4)^3}{(-2 \cdot 3)^3 \cdot (3 \cdot 3)^3 \cdot 5^2} = \frac{(-2)^2 \cdot 3^2 \cdot 2^2 \cdot \cancel{5^2} \cdot \cancel{3^3} \cdot 4^3}{(-2)^3 \cdot \cancel{3^3} \cdot 3^3 \cdot 3^3 \cdot \cancel{5^2}} =$

$= \frac{2^2 \cdot 3^2 \cdot 2^2 \cdot \left(2^2\right)^3}{-2^3 \cdot 3^3 \cdot 3^3} = -\frac{2^2 \cdot \cancel{3^2} \cdot 2^2 \cdot 2^{\cancel{6}\,3}}{\cancel{2^3} \cdot 3^3 \cdot \cancel{3^3}^{\,1}} = -\frac{2^7}{3^4} = -\frac{128}{81}$

22 (a) $\frac{2 \cdot x^{-3} \cdot y^2}{8 \cdot x \cdot y} = \frac{2 \cdot y^2}{8x^3 \cdot x \cdot y} = \frac{y}{4x^4}$

(b) $\frac{15 \cdot s^{-2} \cdot t^3}{45 \cdot s \cdot t^{-2}} = \frac{1 \cdot t^3 \cdot t^2}{3 \cdot s^2 \cdot s} = \frac{t^5}{3s^3}$

(c) $4 \cdot u \cdot \frac{1}{12 \cdot u^{-2}} = \frac{4u \cdot u^2}{12} = \frac{u^3}{3}$

(d) $\frac{48 \cdot x^3 \cdot y}{x^2 \cdot y^{-3}} \cdot 4^{-3} \cdot x^{-4} \cdot y^{-5} = \frac{48x^3y \cdot y^3}{x^2} \cdot \frac{1}{4^3 \cdot x^4y^5} = \frac{48x^3y^4}{64x^6y^5} = \frac{3}{4x^3y}$

(e) $\frac{3 \cdot x^{-2}}{y^{-3} \cdot x} : \frac{12 \cdot x^{-3} \cdot y^0}{x^2 \cdot y^{-4}} = \frac{3y^3}{x^2 \cdot x} : \frac{12y^4 \cdot 1}{x^2 \cdot x^3} = \frac{3y^3}{x^3} \cdot \frac{x^5}{12y^4} = \frac{x^2}{4y}$

(f) $\frac{7^{-2} \cdot b^{-3}}{3^2 \cdot a^{-2} \cdot b} : \frac{3^{-3} \cdot c^{-2} \cdot a^{-3}}{49 \cdot b^2 \cdot c} = \frac{a^2}{3^2 \cdot 7^2 \cdot b^3 \cdot b} : \frac{1}{3^3 \cdot 49 \cdot c^2 \cdot b^2 \cdot a^3 \cdot c} = \frac{a^2}{3^2 \cdot 7^2 \cdot b^4} \cdot 3^3 \cdot 49c^3 \cdot b^2 \cdot a^3 = \frac{3a^5c^3}{b^2}$

23 (a) $\left(3^{-2}\right)^3 = \frac{1}{3^6} = \frac{1}{729}$

(b) $\left[(-4)^0\right]^4 = 1^4 = 1$

(c) $\left(2^{-2}\right)^{-3} = 2^6 = 64$

(d) $\left(4 \cdot x^{-3}\right)^{-2} = 4^{-2} \cdot x^6 = \frac{x^6}{4^2} = \frac{x^6}{16}$

(e) $\left(-3^3 \cdot a^{-2}\right)^{-1} = (-3)^{-3} \cdot a^2 = -\frac{a^2}{3^3} = -\frac{a^2}{27}$

(f) $\left(-5 \cdot x^2 \cdot y^{-3}\right)^{-2} = (-5)^{-2} \cdot x^{-4} \cdot y^6 = \frac{y^6}{5^2x^4} = \frac{y^6}{25x^4}$

(g) $5 \cdot \left(-x^{-2} \cdot y^{-2}\right)^{-3} = 5 \cdot (-1)^{-3} \cdot x^6 \cdot y^6 = 5 \cdot \frac{1}{(-1)^3} x^6y^6 = -5x^6y^6$

(h) $x^{-3} \cdot \left(x^{-2} \cdot y^{-3}\right)^{-2} \cdot x^2 = x^{-3} \cdot x^4y^6 \cdot x^2 = x^3y^6$

(i) $3 \cdot x^2 \cdot \left(3^{-2} \cdot x^0 \cdot y^{-3}\right)^{-3} \cdot x^{-1} = 3x^2 \cdot 3^6 \cdot x^0 \cdot y^9 \cdot x^1 = 3^7 \cdot x^3y^9 = 2\,187x^3y^9$

24 (a) $\left(\frac{x}{y}\right)^{-3} = \left(\frac{y}{x}\right)^3 = \frac{y^3}{x^3}$

(b) $\left(\frac{2}{3}\right)^{-3} \cdot \left(\frac{3}{4}\right)^{-1} \cdot \left(\frac{3}{8}\right)^0 = \left(\frac{3}{2}\right)^3 \cdot \frac{4}{3} \cdot 1 = \frac{3^3 \cdot 4}{2^3 \cdot 3} = \frac{3^2}{2} = \frac{9}{2}$

(c) $\left(-\frac{2}{5}\right)^{-2} \cdot \left(\frac{2}{5}\right)^3 \cdot \left(\frac{5}{4}\right)^0 = \left(-\frac{5}{2}\right)^2 \cdot \left(\frac{2}{5}\right)^3 \cdot 1 = \frac{5^2}{2^2} \cdot \frac{2^3}{5^3} = \frac{2}{5}$

(d) $\left(a^2 \cdot b^{-3}\right)^{-2} \cdot \left(a^{-3} \cdot b^4\right)^{-1} = a^{-4}b^6 \cdot a^3 \cdot b^{-4} = a^{-1} \cdot b^2 = \frac{b^2}{a}$

(e) $\left(-3 \cdot x^2 \cdot y^{-2}\right)^4 \cdot \left(-9 \cdot x^2 \cdot y^{-1}\right)^{-3} = (-3)^4 \cdot x^8 \cdot y^{-8} \cdot (-9)^{-3} \cdot x^{-6} \cdot y^3 =$

$= (-3)^4 \cdot x^2 \cdot y^{-5} \cdot \frac{1}{(-9)^3} = \frac{3x^4x^2}{-9^3y^5} = -\frac{3^4x^2}{3^6y^5} = -\frac{x^2}{3^2y^5} = -\frac{x^2}{9y^5}$

(f) $\left(-2 \cdot x^5 \cdot y^{-1}\right)^{-2} \cdot \left(4 \cdot x^2 \cdot y^0\right)^3 = (-2)^{-2} \cdot x^{-10}y^2 \cdot 4^3x^6 \cdot 1 = \frac{1}{(-2)^2} \cdot x^{-4}y^2 \cdot 2^6 = \frac{y^2 \cdot 2^6}{2^2\,x^4} = \frac{2^4 \cdot y^2}{x^4} = \frac{16y^2}{x^4}$

25 (a) $\left(a^2b^{-3}\right)^{-4} \cdot \left(a^{-2}b^4\right)^{-3} = a^{-8} \cdot b^{12} \cdot a^6 \cdot b^{-12} = a^{-2} \cdot b^0 = \frac{1}{a^2}$

(b) $\left(-3a^{-2}b^3\right)^3 \cdot \left(9a^{-3}b^4\right)^{-2} = (-3)^3 \cdot a^{-6} \cdot b^9 \cdot 9^{-2} \cdot a^6 \cdot b^{-8} = (-3)^3 \cdot \frac{1}{9^2} \cdot a^0 \cdot b = -\frac{27b}{81} = -\frac{b}{3}$

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26 (a) $\frac{4x^4y^{-3}}{5z^{-2}x^{-3}} : \frac{16\cdot(xz)^{-4}y^{-2}}{15\left(x^{-2}z\right)^3y^{-3}} = \frac{4x^4z^2x^3}{5y^3} : \frac{16\cdot x^{-4}\cdot z^{-4}\cdot y^3}{15x^{-6}\cdot z^3\cdot y^2} = \frac{4x^7z^2}{5y^3} : \frac{4^2\cdot x^6\cdot y^3}{15x^4\cdot z^4\cdot z^3\cdot y^2} = \frac{4x^7z^2}{5y^3} : \frac{4^2\cdot x^2y}{15z^7} =$

$= \frac{4x^7z^2}{5y^3}\cdot\frac{3\cdot 5\cdot z^7}{4^2\cdot x^2y} = \frac{3x^5z^9}{4y^4}$

(b) $\frac{7a^{-3}b^4}{9c^{-5}a^4} : \frac{27^{-1}\left(bc^{-3}\right)^{-2}}{14^{-3}\left(ab^2\right)^{-3}} = \frac{7b^4c^5}{9a^3a^4} : \frac{14^3\left(ab^2\right)^3}{27\left(bc^{-3}\right)^2} = \frac{7b^4c^5}{9a^3a^4} : \frac{(2\cdot 7)^3\cdot a^3b^6}{3^3\cdot b^2\cdot c^{-6}} = \frac{7b^4c^5}{9a^3a^4}\cdot\frac{3^3\cdot b^2}{2^3\cdot 7^3\cdot a^3b^6c^6} = \frac{7\cdot 3^3\cdot b^6c^5}{3^2\cdot 2^3\cdot 7^3\cdot a^{10}b^6c^6} = \frac{3}{2^3\cdot 7^2\cdot a^{10}c} = \frac{3}{329a^{10}c}$

27 (a) $\left(\frac{3}{2}\right)^{-2} : \left(\frac{3}{2}\right)^3 = \left(\frac{3}{2}\right)^{-2-3} = \left(\frac{3}{2}\right)^{-5} = \left(\frac{2}{3}\right)^5 = \frac{2^5}{3^5} = \frac{32}{243}$

(b) $\left(-\frac{1}{3}\right)^6 : \left(-\frac{1}{3}\right)^3 = \left(-\frac{1}{3}\right)^{6-3} = \left(-\frac{1}{3}\right)^3 = -\frac{1}{27}$

(c) $(6b)^{-2} : (9b)^{-3} = \frac{1}{6^2\cdot b^2} : \frac{1}{9^3\cdot b^3} = \frac{1}{6^2\cdot b^2}\cdot 9^3\cdot b^3 = \frac{\left(3^2\right)^3b^3}{(2\cdot 3)^2\cdot b^2} = \frac{3^6\cdot b^3}{2^2\cdot 3^2\cdot b^2} = \frac{3^4\cdot b}{2^2} = \frac{81b}{4}$

(d) $\left(\frac{3}{7}\right)^{-3}\cdot\left(\frac{14}{9}\right)^{-2} = \left(\frac{7}{3}\right)^3\cdot\left(\frac{9}{14}\right)^2 = \frac{7^3}{3^3}\cdot\frac{3^4}{2^2\cdot 7^2} = \frac{7\cdot 3}{2^2} = \frac{21}{4}$

(e) $\left(\frac{5x}{3}\right)^{-2}\cdot\left(\frac{9^{-2}x^2}{25^{-1}}\right)^3 = \left(\frac{3}{5x}\right)^2\cdot\frac{9^{-6}\cdot x^6}{25^{-3}} = \frac{3^2}{5^2\cdot x^2}\cdot\frac{25^3\cdot x^6}{9^6} = \frac{3^2\cdot 5^6\cdot x^6}{5^2\cdot x^2\cdot 3^{12}} = \frac{5^4\cdot x^6}{3^{10}} = \frac{625x^6}{59\,049}$

(f) $\left(-\frac{3a}{2b^{-2}}\right)^{-3}\cdot\left(\frac{2^{-2}a^2}{3^{-1}b^{-1}}\right)^2 = \left(-\frac{2b^{-2}}{3a}\right)\cdot\frac{2^{-4}\cdot a^4}{3^{-2}\cdot b^{-2}} = -\frac{2^3\cdot b^{-6}}{3^3\cdot a^3}\cdot\frac{2^{-4}\cdot a^4}{3^{-2}\cdot b^{-2}} = -\frac{2^3}{3^3\cdot a^3b^6}\cdot\frac{3^2\cdot b^2a^4}{2^4} = -\frac{a}{2\cdot 3b^4} = -\frac{a}{6b^4}$

28 (a) $\left(x^4+x^{-3}+x^{-2}-x\right)\cdot x = x^5+\frac{1}{x^2}+\frac{1}{x}-x^2$

(b) $\left(-x^{-3}+x^{-2}+2x^{-1}-3\right)\cdot x^{-2} = -\frac{1}{x^5}+\frac{1}{x^4}+\frac{2}{x^3}-\frac{3}{x^2}$

(c) $-a^{-3}\cdot\left(a^5-2a^4+3a^{-2}-a^{-1}\right) = -a^2+2a-\frac{3}{a^5}+\frac{1}{a^4}$

(d) $\left(a^{3n-1}+a^{3n}+a^{3n+1}\right)\cdot\left(a^n-1\right) = a^{3n-1+n}+a^{3n+n}+a^{3n+1+n-a^{3n-1}}a^{3n+1} = a^{4n-1}+a^{4n}+a^{4n+1}-a^{3n-1}-a^{3n}-a^{3n+1}$

(e) $\left(a^{2n}-b^{2n}\right)\cdot\left(a^{2n}+b^{2n}\right) = a^{4n}-b^{4n}$

(f) $\left(a^n-b^{n+1}\right)^2 = \left(a^n\right)^2-2\cdot a^n\cdot b^{n+1}+\left(b^{n+1}\right)^2 = a^{2n}-2a^nb^{n+1}+b^{2n+2}$

29 (a) $\frac{a^3}{b^2c} = a^3b^{-2}c^{-1}$

(b) $\frac{a^n}{b^n} = a^nb^{-n}$

(c) $\frac{a^{n+1}}{b^{n+1}} = a^{n+1}b^{-(n+1)} = a^{n+1}b^{-n-1}$

(d) $\frac{a}{b} : a^3 = \frac{a}{b}\cdot\frac{1}{a^3} = \frac{1}{a^2b} = a^{-2}b^{-1}$

(e) $\frac{a^{r+s}}{b^r} : \frac{a^{r-s}}{b^{r+s}} = a^{r+s-(r-s)}\cdot b^{r+s-r} =$
$= a^{r+s-r+s}\cdot b^s = a^{2s}b^s$

(f) $\frac{a^{-2}}{b^3c} : \frac{(ab)^{1-n}}{c^{n+1}} = \frac{a^{-2-(1-n)}\cdot c^{n+1-1}}{b^{3+1-n}} = \frac{a^{-3+n}\cdot c^n}{b^{4-n}} =$
$= a^{-3+n}\cdot b^{n-4}\cdot c^n$

30 (a) $\left(3x^2-2y^3\right)^2 = 9x^4-12x^2y^3+4y^6$

(b) $\left(4u^3+3v^2\right)^3 = (4u)^3+3\cdot\left(4u^3\right)^2\cdot 3v^2+3\cdot\left(4u^3\right)\cdot\left(3v^2\right)^2+\left(3v^2\right)^3 =$
$= 64u^9+3\cdot 16u^6\cdot 3v^2+12u^3\cdot 9v^4+27v^6 = 64u^9+144u^6v^2+108u^3v^2+27v^6$

(c) $\left(a^{2n}-3b^n\right)^2 = \left(a^{2n}\right)^2-2\cdot\left(a^{2n}\right)\cdot\left(3b^n\right)+\left(3b^n\right)^2 = a^{4n}-6a^{2n}b^n+9b^{2n}$

(d) $\left(y^{n-1}+3z^{n+1}\right)^3 = \left(y^{n-1}\right)^3+3\cdot\left(y^{n-1}\right)^2\cdot 3z^{n+1}+3\cdot y^{n-1}\cdot\left(3z^{n+1}\right)^2+\left(3z^{n+1}\right)^3 =$
$= y^{3n-3}+9y^{2n-2}\cdot z^{n+1}+27y^{n-1}z^{2n+2}+27z^{3n+3}$

31 (a) $\left(\frac{1}{a}+a\right)^2 = \left(\frac{1}{a}\right)^2+2\cdot\frac{1}{a}\cdot a+a^2 = \frac{1}{a^2}+2+a^2$

(b) $\left(-\frac{3}{b}+2b\right)^3 = \left(2b-\frac{3}{b}\right)^3 = (2b)^3-3\cdot(2b)^2\cdot\frac{3}{b}+3\cdot 2b\cdot\left(\frac{3}{b}\right)^2-\left(\frac{3}{b}\right)^3 = 8b^3-3\cdot 4b^2\cdot\frac{3}{b}+6b\cdot\frac{9}{b^2}-\frac{27}{b^3} =$
$= 8b^3-36b+\frac{54}{b}-\frac{27}{b^3}$

(c) $\left(a^3-b^{-2}\right)^3 = \left(a^3-\frac{1}{b^2}\right)^3 = \left(a^3\right)^3-3\cdot\left(a^3\right)^2\cdot\frac{1}{b^2}+3a^3\cdot\left(\frac{1}{b^2}\right)^2-\left(\frac{1}{b^2}\right)^3 =$
$= a^9-\frac{3a^6}{b^2}+\frac{3a^3}{b^4}-\frac{1}{b^6}$

(d) $\left(a^{-3}+a^3\right)^3 = \left(\frac{1}{a^3}+a^3\right)^3 = \left(\frac{1}{a^3}\right)^3+3\cdot\left(\frac{1}{a^3}\right)^2\cdot a^3+3\cdot\frac{1}{a^3}\cdot\left(a^3\right)^2+\left(a^3\right)^3 =$
$= \frac{1}{a^9}+3\cdot\frac{1}{a^6}\cdot a^3+3\cdot\frac{1}{a^3}\cdot a^6+a^9 = \frac{1}{a^9}+\frac{3}{a^3}+3a^3+a^9$

32 In 1 s … $3\cdot 10^8$ m

$1\,\text{min} = 60\text{ s} = 6\cdot 10\text{ s}$
$1\,\text{h} = 60\text{ min} = 6\cdot 10\text{ min} = 6\cdot 10\cdot 6\cdot 10\text{ s} = 36\cdot 10^2\text{ s} = 3{,}6\cdot 10^3\text{ s}$
$1\,\text{d} = 24\text{ h} = 24\cdot 3{,}6\cdot 10^3\text{ s} = 86{,}4\cdot 10^3\text{ s} = 8{,}64\cdot 10^4\text{ s}$
$1\,\text{a} = 365\text{ d} = 365\cdot 8{,}64\cdot 10^3\text{ s} = 3153{,}6\cdot 10^4\text{ s} = 3{,}1536\cdot 10^7\text{ s}$

In 1 a … $3\cdot 10^8\cdot 3{,}1536\cdot 10^7\text{ m} = 9{,}4608\cdot 10^{15}$ m

Ein Lichtjahr entspricht der Strecke von $9{,}4608\cdot 10^{15}$ m, das sind rund 10^{16} m.

33 Nach dem 1.Falten: $0{,}01\text{mm}\cdot 2$
Nach dem 2.Falten: $(0{,}01\text{mm}\cdot 2)\cdot 2 = 0{,}01\cdot 2^2$ mm
Nach dem 3.Falten: $(0{,}01\cdot 2^2\text{ mm})\cdot 2 = 0{,}01\cdot 2^3$ mm
.
.
Nach dem 50.Falten: $0{,}01\cdot 2^{50}$ mm
$0{,}01\cdot 2^{50}\text{ mm} = 1{,}1258990..\cdot 10^{13}$ mm
Es würde rund $1{,}1259\cdot 10^{13}$ mm dick sein.
$1{,}1259\cdot 10^{13}\text{ mm} = 1{,}1259\cdot 10^{10}$ m
$1{,}1259\cdot 10^{10}\text{ m} = 1{,}1259\cdot 10^{7}$ km
Es würde rund $1{,}1259\cdot 10^{7}$ km dick sein.

34 $K_n = 1\cdot\left(1+\frac{2}{100}\right)^{2015} = 2{,}1347..\cdot 10^{17}$
Man hätte im Jahr 2015 rund $2{,}1347\cdot 10^{17}$ € (das wären 213,47 Billiarden €)

35 $1{,}7\cdot 10^9$ … 20% des Jahres
$6\cdot 10^9$ … x% des Jahres

$x = 6\cdot 10^9\cdot 0{,}2 : (1{,}7\cdot 10^9) = 0{,}70588.. \mathrel{\hat{=}} 70{,}588..\%$
70,588..% des Jahres entsprechen 70,588.. · 12 = 8,47.. Monate
Mit 6 Milliarden m^3 Erdgas kommt man durchschnittlich 8,5 Monate aus.

36 (a) $\sqrt{3} = 3^{\frac{1}{2}}$ (b) $\sqrt[3]{2^2} = 2^{\frac{2}{3}}$

(c) $\sqrt[4]{3x^2} = \left(3x^2\right)^{\frac{1}{4}} = 3^{\frac{1}{4}}\cdot x^{\frac{2}{4}} = 3^{\frac{1}{4}}\cdot x^{\frac{1}{2}}$ (d) $\sqrt{\frac{1}{x}} = \left(\frac{1}{x}\right)^{\frac{1}{2}}$

(e) $\sqrt[n]{\frac{1}{(x-y)^2}} = \left[\frac{1}{(x-y)^2}\right]^{\frac{1}{n}} = \frac{1^{\frac{1}{n}}}{(x-y)^{\frac{2}{n}}} = \frac{1}{(x-y)^{\frac{2}{n}}}$

(f) $\sqrt[4]{\left(\frac{x-y}{x^2-y^2}\right)^3} = \left(\frac{x-y}{x^2-y^2}\right)^{\frac{3}{4}} = \left[\frac{x-y}{(x-y)(x+y)}\right]^{\frac{3}{4}} = \left(\frac{1}{x+y}\right)^{\frac{3}{4}}$

(g) $\frac{3}{\sqrt[3]{x^2+y^2}} = \frac{3}{\left(x^2+y^2\right)^{\frac{1}{3}}}$

(h) $\frac{\sqrt[3]{x^2-y^2}}{\sqrt{x^2-y^2}} = \frac{\left(x^2-y^2\right)^{\frac{1}{3}}}{\left(x^2-y^2\right)^{\frac{1}{2}}} = \left(x^2-y^2\right)^{\frac{1}{3}-\frac{1}{2}} = \left(x^2-y^2\right)^{\frac{2}{6}-\frac{3}{6}} = \left(x^2-y^2\right)^{-\frac{1}{6}}$

37 (a) $3^{\frac{1}{2}} = \sqrt{3}$ (b) $7^{\frac{1}{3}} = \sqrt[3]{7}$

(c) $4^{\frac{3}{4}} = \sqrt[4]{4^3} = \sqrt[4]{64}$ (d) $(4x)^{-\frac{1}{2}} = \left(\frac{1}{4x}\right)^{\frac{1}{2}} = \sqrt{\frac{1}{4x}} = \frac{1}{2\sqrt{x}}$

(e) $\left(5ab^2\right)^{-\frac{2}{3}} = \sqrt[3]{\left(\frac{1}{5ab^2}\right)^2} = \sqrt[3]{\frac{1}{25a^2b^4}} = \frac{1}{\sqrt[3]{25a^2b^4}}$ (f) $7xy^{-\frac{3}{4}} = 7x\cdot\left(\frac{1}{y}\right)^{\frac{3}{4}} = 7x\cdot\sqrt[4]{\left(\frac{1}{y}\right)^3} = 7x\cdot\sqrt[4]{\frac{1}{y^3}} = \frac{7x}{\sqrt[4]{y^3}}$

(g) $3\cdot\left(2x^{\frac{2}{3}}\right)^{-1} = 3\cdot\frac{1}{2x^{\frac{2}{3}}} = \frac{3}{2}\cdot\frac{1}{\sqrt[3]{x^2}}$ (h) $\frac{1}{2}\cdot\left(3x^{-2}\right)^{\frac{1}{3}} = \frac{1}{2}\cdot\left(\frac{3}{x^2}\right)^{\frac{1}{3}} = \frac{1}{2}\cdot\sqrt[3]{\frac{3}{x^2}}$

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38 (a) $\sqrt{256}=\sqrt{16^2}=16$ (b) $\sqrt[3]{0{,}008}=\sqrt[3]{0{,}2^3}=0{,}2$

(c) $\sqrt[3]{0{,}343}=\sqrt[3]{0{,}7^3}=0{,}7$ (d) $\sqrt[5]{0{,}00001}=\sqrt[5]{0{,}1^5}=0{,}1$

(e) $\sqrt[4]{\frac{16}{81}}=\sqrt[4]{\frac{2^4}{3^4}}=\frac{2}{3}$ (f) $\sqrt[3]{0{,}125}=\sqrt[3]{0{,}5^3}=0{,}5$

39 (a) $\sqrt{2}\cdot\sqrt{18}=\sqrt{2}\cdot\sqrt{2\cdot 9}=\sqrt{2}\cdot\sqrt{2}\cdot\sqrt{9}=2\cdot 3=6$ (b) $\sqrt{7}\cdot\sqrt{63}=\sqrt{7}\cdot\sqrt{7\cdot 9}=7\cdot\sqrt{9}=7\cdot 3=21$

(c) $\sqrt[4]{27}\cdot\sqrt[4]{3}=\sqrt[4]{3^3}\cdot\sqrt[4]{3}=\sqrt[4]{3^4}=3$ (d) $\sqrt[6]{4}\cdot\sqrt[6]{16}=\sqrt[6]{2^2}\cdot\sqrt[6]{2^4}=\sqrt[6]{2^6}=2$

(e) $\sqrt[3]{30}\cdot\sqrt[3]{20}\cdot\sqrt[3]{45}=\sqrt[3]{2\cdot 3\cdot 5}\cdot\sqrt[3]{2\cdot 2\cdot 5}\cdot\sqrt[3]{3\cdot 3\cdot 5}=\sqrt[3]{2^3\cdot 3^3\cdot 5^3}=2\cdot 3\cdot 5=30$

(f) $\sqrt[4]{\frac{4}{25}}\cdot\sqrt[4]{\frac{54}{5}}\cdot\sqrt[4]{\frac{6}{5}}=\sqrt[4]{\frac{2^2}{5^2}}\cdot\sqrt[4]{\frac{2\cdot 3^3}{5}}\cdot\sqrt[4]{\frac{2\cdot 3}{5}}=\sqrt[4]{\frac{2^4}{5^4}}=\frac{2\cdot 3}{5}=\frac{6}{5}$

40 (a) $5^2\cdot 5^{\frac{1}{2}}=5^{2+\frac{1}{2}}=5^{\frac{5}{2}}$ (b) $3^{-\frac{1}{3}}\cdot 3^2=3^{\frac{5}{3}}$

(c) $4^{-\frac{1}{2}}\cdot 16=4^{-\frac{1}{2}}\cdot 4^2=4^{\frac{3}{2}}=\left(2^2\right)^{\frac{3}{2}}=2^{2\cdot\frac{3}{2}}=2^3$ (d) $0{,}04^{-\frac{1}{2}}\cdot 0{,}2^3=\left(0{,}2^2\right)^{-\frac{1}{2}}\cdot 0{,}2^3=0{,}2^{-\frac{2}{2}}\cdot 0{,}2^3=0{,}2^2$

(e) $5^{\frac{1}{2}}:5^{\frac{1}{3}}=5^{\frac{1}{2}-\frac{1}{3}}=5^{\frac{3}{6}-\frac{2}{6}}=5^{\frac{1}{6}}$ (f) $7^0:49^{-\frac{1}{3}}=7^0:\left(7^2\right)^{-\frac{1}{3}}=7^0:7^{-\frac{2}{3}}=7^{0-\left(-\frac{2}{3}\right)}=7^{\frac{2}{3}}$

(g) $\left(\frac{1}{4}\right)^{\frac{1}{3}}:\left(\frac{1}{16}\right)^{\frac{1}{4}}=\left(\frac{1}{4}\right)^{\frac{1}{3}}:\left[\left(\frac{1}{4}\right)^2\right]^{\frac{1}{4}}=\left(\frac{1}{4}\right)^{\frac{1}{3}}:\left(\frac{1}{4}\right)^{\frac{2}{4}}=\left(\frac{1}{4}\right)^{\frac{1}{3}}:\left(\frac{1}{4}\right)^{\frac{1}{2}}=\left(\frac{1}{4}\right)^{\frac{1}{3}-\frac{1}{2}}=\left(\frac{1}{4}\right)^{\frac{2}{6}-\frac{3}{6}}=\left(\frac{1}{4}\right)^{-\frac{1}{6}}=4^{\frac{1}{6}}$

(h) $\left(\frac{1}{5}\right)^{-\frac{1}{2}}:\left(\frac{1}{5}\right)^{\frac{1}{3}}=\left(\frac{1}{5}\right)^{-\frac{1}{2}-\frac{1}{3}}=\left(\frac{1}{5}\right)^{-\frac{3}{6}-\frac{2}{6}}=\left(\frac{1}{5}\right)^{-\frac{5}{6}}=5^{\frac{5}{6}}$

41 (a) $\left(3^2\right)^{\frac{1}{2}}=3^{2\cdot\frac{1}{2}}=3$ (b) $\left(8^{-\frac{1}{3}}\right)^0=1$

(c) $\left(5^{\frac{3}{4}}\right)^{-4}=5^{\frac{3}{4}\cdot(-4)}=5^{-3}=\frac{1}{5^3}=\frac{1}{125}$ (d) $\left(3^{-0{,}5}\right)^2=3^{-0{,}5\cdot 2}=3^{-1}=\frac{1}{3}$

42 (a) $\left(3^{\frac{1}{2}}\right)^6=3^{\frac{1}{2}\cdot 6}=3^{\frac{6}{2}}=3^3=27$ (b) $\left(7^{-3}\right)^{-\frac{2}{3}}=7^{-3\cdot\left(-\frac{2}{3}\right)}=7^{\frac{6}{3}}=7^2=49$

(c) $\left(4{,}5^{-4}\right)^{\frac{1}{4}}=4{,}5^{-4\cdot\frac{1}{4}}=4{,}5^{-1}=\left(\frac{9}{2}\right)^{-1}=\frac{2}{9}$

(d) $\left(0{,}2^{-3}\right)^{\frac{2}{3}}=0{,}2^{-3\cdot\frac{2}{3}}=0{,}2^{-2}=\left(\frac{2}{10}\right)^{-2}=\left(\frac{10}{2}\right)^2=5^2=25$

43 (a) $3^{\frac{1}{4}}\cdot\left(\frac{1}{27}\right)^{\frac{1}{4}}=\left(3\cdot\frac{1}{27}\right)^{\frac{1}{4}}=\left(\frac{1}{9}\right)^{\frac{1}{4}}=\left[\left(\frac{1}{3}\right)^2\right]^{\frac{1}{4}}=\left(\frac{1}{3}\right)^{\frac{2}{4}}=\left(\frac{1}{3}\right)^{\frac{1}{2}}=\sqrt{\frac{1}{3}}=\frac{1}{\sqrt{3}}$

(b) $7^{-\frac{1}{2}}\cdot 343^{\frac{1}{2}}=\left(\frac{1}{7}\right)^{\frac{1}{2}}\cdot 343^{\frac{1}{2}}=\left(\frac{1}{7}\cdot 343\right)^{\frac{1}{2}}=\left(49\right)^{\frac{1}{2}}=\sqrt{49}=7$

(c) $\left(\frac{4}{3}\right)^{-\frac{1}{2}}\cdot 2{,}25^{-\frac{1}{2}}=\left(\frac{4}{3}\cdot 2{,}25\right)^{-\frac{1}{2}}=\left(\frac{4}{3}\cdot\frac{9}{4}\right)^{-\frac{1}{2}}=3^{-\frac{1}{2}}=\left(\frac{1}{3}\right)^{\frac{1}{2}}=\sqrt{\frac{1}{3}}=\frac{1}{\sqrt{3}}$

(d) $\left(\frac{5}{4}\right)^{-\frac{1}{2}}\cdot 6{,}25^{\frac{1}{2}}=\left(\frac{4}{5}\right)^{\frac{1}{2}}\cdot 6{,}25^{\frac{1}{2}}=\left(\frac{4}{5}\cdot 6{,}25\right)^{\frac{1}{2}}=\left(\frac{4}{5}\cdot\frac{25}{4}\right)^{\frac{1}{2}}=5^{\frac{1}{2}}=\sqrt{5}$

44 (a) $\left(\frac{1}{2}\right)^{\frac{1}{2}}\cdot\left(\frac{1}{18}\right)^{\frac{1}{2}}=\left(\frac{1}{2}\cdot\frac{1}{18}\right)^{\frac{1}{2}}=\left(\frac{1}{36}\right)^{\frac{1}{2}}=\sqrt{\frac{1}{36}}=\frac{1}{6}$

(b) $63^{\frac{1}{2}}\cdot\left(\frac{7}{4}\right)^{\frac{1}{2}}=\left(63\cdot\frac{7}{4}\right)^{\frac{1}{2}}=\left(\frac{441}{4}\right)^{\frac{1}{2}}=\sqrt{\frac{441}{4}}=\frac{21}{2}$

(c) $\left(\frac{4}{x^2}\right)^{\frac{1}{6}}\cdot\left(16x^2\right)^{\frac{1}{6}}=\left(\frac{4}{x^2}\cdot 16x^2\right)^{\frac{1}{6}}=64^{\frac{1}{6}}=\left(2^6\right)^{\frac{1}{6}}=2$

(d) $\left(\frac{3x^3}{8y}\right)^{\frac{1}{8}}\cdot\left(\frac{27x^4}{4y^4}\right)^{\frac{1}{8}}\cdot\left(\frac{81x}{8y^3}\right)^{\frac{1}{8}}=\left(\frac{3\cdot 27\cdot 81\cdot x^8}{8\cdot 4\cdot 8\cdot y^8}\right)^{\frac{1}{8}}=\left(\frac{3^8\cdot x^8}{2^3\cdot 2^2\cdot 2^3\cdot y^8}\right)^{\frac{1}{8}}=\left(\frac{3^8\cdot x^8}{2^8\cdot y^8}\right)^{\frac{1}{8}}=\frac{3x}{2y}$

45 (a) $\sqrt[3]{\frac{ab}{x}}\cdot\sqrt[3]{\frac{a^2x}{b}}=\sqrt[3]{\frac{ab}{x}\cdot\frac{a^2x}{b}}=\sqrt[3]{a^3}=a$

(b) $\sqrt{\frac{36xz}{125y^3}}\cdot\sqrt{\frac{40x^3}{147y^2}}\cdot\sqrt{\frac{24z}{25y}}=\sqrt{\frac{36xz}{125y^3}\cdot\frac{40x^3}{147y^2}\cdot\frac{24z}{25y}}=\sqrt{\frac{36\cdot 40\cdot 24\cdot x^4z^2}{125\cdot 147\cdot 25\cdot y^4}}=\sqrt{\frac{2^2\cdot 3^2\cdot 2^3\cdot 5\cdot 2^3\cdot 3\cdot x^4z^2}{5^3\cdot 3\cdot 7^2\cdot 5^2\cdot y^6}}=\sqrt{\frac{2^8\cdot 3^3\cdot 5\cdot x^4z^2}{3\cdot 5^5\cdot 7^2\cdot y^6}}=$

$=\sqrt{\frac{2^8\cdot 3^2\cdot x^4z^2}{5^4\cdot 7^2\cdot y^6}}=\frac{2^4\cdot 3\cdot x^2z}{5^2\cdot 7\cdot y^3}=\frac{48x^2z}{175y^3}$

(c) $\sqrt[4]{27x^3b^6}\cdot\sqrt[4]{3xb^2}=\sqrt[4]{27x^3b^6\cdot 3xb^2}=\sqrt[4]{3^3\cdot 3\cdot b^8\cdot x^4}=\sqrt[4]{3^4\cdot b^8\cdot x^4}=3b^2x$

(d) $\sqrt[5]{2x^6}\cdot\sqrt[5]{\frac{x^4}{64}}=\sqrt[5]{2x^6\cdot\frac{x^4}{64}}=\sqrt[5]{\frac{x^{10}}{32}}=\sqrt[5]{\frac{x^{10}}{2^5}}=\frac{x^2}{2}$

(e) $\sqrt[6]{\frac{y^2}{x^5}}\cdot\sqrt[6]{16y^3}\cdot\sqrt[6]{\frac{4y}{x}}=\sqrt[6]{\frac{y^2}{x^5}\cdot 16y^3\cdot\frac{4y}{x}}=\sqrt[6]{\frac{y^2\cdot 2^4\cdot y^3\cdot 2^2\cdot y}{x^6}}=\sqrt[6]{\frac{2^6\cdot y^6}{x^6}}=\frac{2y}{x}$

(f) $\sqrt[5]{\frac{54a^2x^7}{25b^{12}}}\cdot\sqrt[5]{\frac{7x^{11}}{25b^{10}a^3}}\cdot\sqrt[5]{\frac{9b^7a^6}{70x^8}}=\sqrt[5]{\frac{54a^2x^7\cdot 7x^{11}\cdot 9b^7a^6}{25b^{12}\cdot 25b^{10}a^3\cdot 70x^8}}=\sqrt[5]{\frac{2\cdot 3^3\cdot 7\cdot 3^2\cdot x^{18}\cdot b^7a^8}{5^2\cdot 5^2\cdot 2\cdot 5\cdot 7\cdot x^8\cdot b^{22}\cdot a^3}}=\sqrt[5]{\frac{3^5\cdot x^{10}\cdot a^5}{5^5\cdot b^{15}}}=\frac{3ax^2}{5b^3}$

46 (a) $39^{-\frac{1}{4}}:13^{-\frac{1}{4}}=(39:13)^{-\frac{1}{4}}=3^{-\frac{1}{4}}=\left(\frac{1}{3}\right)^{\frac{1}{4}}=\sqrt[4]{\frac{1}{3}}$

(b) $15^{\frac{1}{2}}:60^{\frac{1}{2}}=(15:60)^{\frac{1}{2}}=\left(\frac{1}{4}\right)^{\frac{1}{2}}=\sqrt{\frac{1}{4}}=\frac{1}{2}$

(c) $\left(\frac{36}{25}\right)^{-\frac{3}{5}}:27^{-\frac{3}{5}}=\left(\frac{36}{25}\cdot\frac{1}{27}\right)^{-\frac{3}{5}}=\left(\frac{4}{25\cdot 3}\right)^{-\frac{3}{5}}=\left(\frac{4}{75}\right)^{-\frac{3}{5}}=\left(\frac{75}{4}\right)^{\frac{3}{5}}=\sqrt[5]{\frac{75^3}{4^3}}=\sqrt[5]{\frac{\left(3\cdot 5^2\right)^3}{\left(2^2\right)^3}}=\sqrt[5]{\frac{3^3\cdot 5^6}{2^6}}=\frac{5}{2}\cdot\sqrt[5]{\frac{3^3\cdot 5}{2}}=\frac{5}{2}\cdot\sqrt[5]{\frac{135}{2}}$

(d) $\frac{\sqrt{3}}{\sqrt{27}}=\sqrt{\frac{3}{27}}=\sqrt{\frac{1}{9}}=\frac{1}{3}$

(e) $\frac{\sqrt[3]{x^n}}{\sqrt[3]{x^{n-3}}}=\sqrt[3]{x^{n-(n-3)}}=\sqrt[3]{x^3}=x$

(f) $\frac{\sqrt[4]{x^{7n}}}{\sqrt[4]{x^{3n}}}=\sqrt[4]{\frac{x^{7n}}{x^{3n}}}=\sqrt[4]{x^{7n-3n}}=\sqrt[4]{x^{4n}}=x^n$

(g) $\frac{\sqrt[3]{x^4y}}{x\cdot\sqrt[3]{xy}}=\frac{\sqrt[3]{x^4y}}{\sqrt[3]{x^4y}}=1$

(h) $\frac{b^2\cdot\sqrt[5]{64a^5bc^2}}{\sqrt[5]{2bc^7}}=\frac{b^2\cdot\sqrt[5]{2^6a^5bc^2}}{\sqrt[5]{2bc^7}}=\frac{b^2\cdot 2a\cdot\sqrt[5]{2bc^2}}{c\cdot\sqrt[5]{2bc^2}}=\frac{2ab^2}{c}$

47 (a) $\sqrt{\sqrt{3}}=\sqrt[4]{3}$

(b) $\sqrt[5]{\sqrt{16}}=\sqrt[5]{4}$

(c) $\sqrt{\sqrt[3]{4}}=\sqrt[6]{4}=\sqrt[2\cdot 3]{2^2}=\sqrt[3]{2}$

(d) $\sqrt[3]{6\cdot\sqrt{6}}=\sqrt[3]{\sqrt{6^2\cdot 6}}=\sqrt[6]{6^3}=\sqrt[2\cdot 3]{6^3}=\sqrt{6}$

(e) $\sqrt{4\cdot\sqrt[3]{4}}=\sqrt{\sqrt[3]{4^3\cdot 4}}=\sqrt[6]{4^4}=\sqrt[6]{\left(2^2\right)^4}=\sqrt[6]{2^8}=2\cdot\sqrt[6]{2^2}=2\cdot\sqrt[2\cdot 3]{2^2}=2\sqrt[3]{2}$

(f) $\sqrt{x\cdot\sqrt{x}}=\sqrt{\sqrt{x^3}}=\sqrt[4]{x^3}$

48 (a) $3\cdot\sqrt{2}=\sqrt{3^2\cdot 2}=\sqrt{18}$

(b) $4\cdot\sqrt[3]{\frac{1}{4}}=\sqrt[3]{4^3\cdot\frac{1}{4}}=\sqrt[3]{4^2}=\sqrt[3]{16}$

(c) $5\cdot\sqrt{0{,}2}=\sqrt{5^2\cdot 0{,}2}=\sqrt{25\cdot 0{,}2}=\sqrt{5}$

(d) $3a\cdot\sqrt{2b}=\sqrt{3^2\cdot a^2\cdot 2b}=\sqrt{18a^2b}$

(e) $3\cdot\sqrt{\frac{2}{3}}=\sqrt{3^2\cdot\frac{2}{3}}=\sqrt{3\cdot 2}=\sqrt{6}$

(f) $2\cdot\sqrt{\frac{3}{4}}=\sqrt{2^2\cdot\frac{3}{4}}=\sqrt{4\cdot\frac{3}{4}}=\sqrt{3}$

49 (a) $\sqrt{50}=\sqrt{25\cdot 2}=5\cdot\sqrt{2}$

(b) $\sqrt{27}=\sqrt{9\cdot 3}=3\cdot\sqrt{3}$

(c) $\sqrt{175}=\sqrt{25\cdot 7}=5\cdot\sqrt{7}$

(d) $3\cdot\sqrt{375}=3\cdot\sqrt{3\cdot 125}=3\cdot\sqrt{3\cdot 5\cdot 25}=$
$=3\cdot 5\cdot\sqrt{3\cdot 5}=15\sqrt{15}$

(e) $\sqrt{24a^2b}=\sqrt{2^3\cdot 3\cdot a^2b}=2a\cdot\sqrt{6b}$

(f) $\sqrt[3]{56x^3y^4}=\sqrt[3]{2^3\cdot 7\cdot x^3y^4}=2xy\cdot\sqrt[3]{7y}$

(g) $\sqrt[4]{32a^4b^5c^2}=\sqrt[4]{2^5\cdot a^4b^5c^2}=2ab\cdot\sqrt[4]{2bc^2}$

(h) $\sqrt[3]{40a^5b^4c^2}=\sqrt[3]{2^3\cdot 5\cdot a^5b^4c^2}=2ab\cdot\sqrt[3]{5a^2bc^2}$

50 (a) $\left(\sqrt{4}\right)^3=2^3=8$

(b) $\left(\sqrt[3]{8}\right)^2=2^2=4$

(c) $\left(\sqrt[3]{64}\right)^2=\left(\sqrt[3]{4^3}\right)^2=4^2=16$

(d) $\left(2\cdot\sqrt[4]{3}\right)^2=4\cdot\sqrt[4]{3^2}=4\cdot\sqrt{3}$

(e) $\left(\sqrt[3]{27x^6y^2}\right)^2=\left(3x^2\cdot\sqrt[3]{y^2}\right)^2=9x^4\cdot\sqrt[3]{y^4}=9x^4y\cdot\sqrt[3]{y}$

(f) $\left(\sqrt[6]{2}\right)^3=\sqrt[6]{2^3}=\sqrt[2\cdot 3]{2^3}=\sqrt{2}$

(g) $\left(\sqrt[2n]{x\cdot y^2z^3}\right)^n=\sqrt[2\cdot n]{\left(x\cdot y^2z^3\right)^n}=\sqrt{xy^2z^3}=yz\cdot\sqrt{xz}$

(h) $\left(\sqrt[n]{x^{2n}y^n}\right)^2=\left(x^2y\right)^2=x^4y^2$

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51 (a) $\left(\sqrt{50}+\sqrt{18}\right)\cdot\sqrt{2}=\sqrt{50}\cdot\sqrt{2}+\sqrt{18}\cdot\sqrt{2}=\sqrt{50\cdot 2}+\sqrt{18\cdot 2}=\sqrt{100}+\sqrt{36}=10+6=16$

(b) $\left(\sqrt{45}-\sqrt{20}\right)\cdot\sqrt{5}=\sqrt{45}\cdot\sqrt{5}-\sqrt{20}\cdot\sqrt{5}=\sqrt{45\cdot 5}-\sqrt{20\cdot 5}=\sqrt{225}-\sqrt{100}=15-10=5$

(c) $\left(\sqrt[3]{72}+\sqrt[3]{9}\right)\cdot 2\sqrt[3]{3}=\sqrt[3]{72}\cdot 2\sqrt[3]{3}+\sqrt[3]{9}\cdot 2\sqrt[3]{3}=2\sqrt[3]{216}+2\sqrt[3]{27}=2\cdot 6+2\cdot 3=18$

(d) $\left(3\sqrt{12}-5\sqrt{75}+3\sqrt{27}\right)\cdot\sqrt{3}=3\sqrt{12}\cdot\sqrt{3}-5\sqrt{75}\cdot\sqrt{3}+3\sqrt{27}\cdot\sqrt{3}=$

$=3\sqrt{36}-5\sqrt{225}+3\sqrt{81}=3\cdot 6-5\cdot 15+3\cdot 9=18-75+27=-30$

(e) $\left(2\sqrt{50}+4\sqrt{98}-\sqrt{338}\right)\cdot 4\sqrt{2}=2\sqrt{50}\cdot 4\sqrt{2}+4\sqrt{98}\cdot 4\sqrt{2}-\sqrt{338}\cdot 4\sqrt{2}=$

$=8\sqrt{50\cdot 2}+16\sqrt{98\cdot 2}-4\sqrt{338\cdot 2}=$

$=8\sqrt{100}+16\sqrt{196}-4\sqrt{676}=$

$=8\cdot 10+16\cdot 14-4\cdot 26=$

$=80+224-104=200$

(f) $\left(3\sqrt[3]{1{,}6}-\sqrt[3]{5{,}4}\right)\cdot\sqrt[3]{40}=3\sqrt[3]{1{,}6}\cdot\sqrt[3]{40}-\sqrt[3]{5{,}4}\cdot\sqrt[3]{40}=3\sqrt[3]{1{,}6\cdot 40}-\sqrt[3]{5{,}4\cdot 40}=$

$=3\sqrt[3]{64}-\sqrt[3]{216}=3\cdot 4-6=12-6=6$

52 (a) $\left(7\sqrt{5}-4\sqrt{3}\right)\cdot\left(7\sqrt{5}+4\sqrt{3}\right)=\left(7\sqrt{5}\right)^2-\left(4\sqrt{3}\right)^2=49\cdot 5-16\cdot 3=245-48=197$

(b) $\left(15+2\sqrt{11}\right)\cdot\left(15-2\sqrt{11}\right)=15^2-\left(2\sqrt{11}\right)^2=225-4\cdot 11=225-44=181$

(c) $\left(6+\sqrt{7}\right)^2+\left(6-\sqrt{7}\right)^2=6^2+2\cdot 6\cdot\sqrt{7}+\left(\sqrt{7}\right)^2+6^2-2\cdot 6\cdot\sqrt{7}+\left(\sqrt{7}\right)^2=$

$=36+12\sqrt{7}+7+36-12\sqrt{7}+7=86$

(d) $\left(4+\sqrt{3}\right)\cdot\left(4-\sqrt{3}\right)=4^2-\left(\sqrt{3}\right)^2=16-3=13$

53 (a) $\left(5+2\sqrt{7}\right)\cdot\left(3-\sqrt{7}\right)=15+6\sqrt{7}-5\sqrt{7}-2\cdot 7=1+\sqrt{7}$

(b) $\sqrt{7+2\sqrt{6}}\cdot\sqrt{7-2\sqrt{6}}=\sqrt{\left(7+2\sqrt{6}\right)\cdot\left(7-2\sqrt{6}\right)}=\sqrt{49-4\cdot 6}=\sqrt{49-24}=\sqrt{25}=5$

(c) $\left(\sqrt{5}+\sqrt{7}-\sqrt{20}\right)\cdot\left(\sqrt{5}+\sqrt{7}\right)=\sqrt{5}\cdot\sqrt{5}+\sqrt{7}\cdot\sqrt{5}-\sqrt{20}\cdot\sqrt{5}+\sqrt{5}\cdot\sqrt{7}+\sqrt{7}\cdot\sqrt{7}-\sqrt{20}\cdot\sqrt{7}=$

$=5+\sqrt{35}-\sqrt{100}+\sqrt{35}+7-\sqrt{140}=5+2\sqrt{35}-10+7-\sqrt{4\cdot 35}=$

$=5+2\sqrt{35}-10+7-2\sqrt{35}=2$

(d) $\left(\sqrt{2}-\sqrt{7}+\sqrt{8}\right)\cdot\left(3\sqrt{2}+\sqrt{7}\right)=\sqrt{2}\cdot 3\sqrt{2}-\sqrt{7}\cdot 3\sqrt{2}+\sqrt{8}\cdot 3\sqrt{2}+\sqrt{2}\cdot\sqrt{7}-\sqrt{7}\cdot\sqrt{7}+\sqrt{8}\cdot\sqrt{7}=$

$=3\cdot 2-3\sqrt{14}+3\sqrt{16}+\sqrt{14}-7+\sqrt{56}=6-2\sqrt{14}+3\cdot 4-7+\sqrt{4\cdot 14}=$

$=6-2\sqrt{14}+12-7+2\sqrt{14}=11$

54 (a) $\left(\sqrt{192}-\sqrt{147}\right):\sqrt{3}=\sqrt{192}:\sqrt{3}-\sqrt{147}:\sqrt{3}=\sqrt{192:3}-\sqrt{147:3}=\sqrt{64}-\sqrt{49}=8-7=1$

(b) $\left(\sqrt{20}+2\sqrt{605}-\sqrt{405}\right):\sqrt{5}=\sqrt{20}:\sqrt{5}+2\sqrt{605}:\sqrt{5}-\sqrt{405}:\sqrt{5}=\sqrt{20:5}+2\sqrt{605:5}-\sqrt{405:5}=$

$=\sqrt{4}+2\sqrt{121}-\sqrt{81}=2+2\cdot 11-9=2+22-9=15$

(c) $\left(\sqrt[3]{32}+3\sqrt[3]{500}-7\sqrt[3]{256}\right):\sqrt[3]{4}=\sqrt[3]{32}:\sqrt[3]{4}+3\sqrt[3]{500}:\sqrt[3]{4}-7\sqrt[3]{256}:\sqrt[3]{4}=$

$=\sqrt[3]{8}+3\sqrt[3]{125}-7\sqrt[3]{64}=2+3\cdot 5-7\cdot 4=2+15-28=-11$

55 (a) $\sqrt{8}+\sqrt{32}+\sqrt{16}=\sqrt{2\cdot 4}+\sqrt{2\cdot 16}+4=2\sqrt{2}+4\cdot\sqrt{2}+4=4+6\sqrt{2}$

(b) $2\sqrt{44}+\sqrt{99}-3\sqrt{176}=2\sqrt{4\cdot 11}+\sqrt{9\cdot 11}-3\sqrt{11\cdot 16}=2\cdot 2\sqrt{11}+3\sqrt{11}-3\cdot 4\sqrt{11}=$

$=4\sqrt{11}+3\sqrt{11}-12\sqrt{11}=-5\sqrt{11}$

(c) $3\sqrt{18}+5\sqrt{162}-\sqrt{8}-2\sqrt{72}-4\sqrt{98}=3\sqrt{2\cdot 9}+5\sqrt{2\cdot 81}-\sqrt{2\cdot 4}-2\sqrt{2\cdot 36}-4\sqrt{2\cdot 49}=$

$=3\cdot 3\sqrt{2}+5\cdot 9\sqrt{2}-2\sqrt{2}-2\cdot 6\sqrt{2}-4\cdot 7\sqrt{2}=9\sqrt{2}+45\sqrt{2}-2\sqrt{2}-12\sqrt{2}-28\sqrt{2}=12\sqrt{2}$

(d) $2\sqrt[3]{432}-3\sqrt[3]{54}+4\sqrt[3]{128}-5\sqrt[3]{250}=2\sqrt[3]{2\cdot 216}-3\sqrt[3]{2\cdot 27}+4\sqrt[3]{2\cdot 64}-5\sqrt[3]{2\cdot 125}=$

$=2\cdot 6\sqrt[3]{2}-3\cdot 3\sqrt[3]{2}+4\cdot 4\sqrt[3]{2}-5\cdot 5\sqrt[3]{2}=12\sqrt[3]{2}-9\sqrt[3]{2}+16\sqrt[3]{2}-25\sqrt[3]{2}=-6\sqrt[3]{2}$

56 (a) $\frac{1}{xy^2}\cdot\sqrt{\frac{x^3y}{2}}=\sqrt{\frac{1}{x^2y^4}\cdot\frac{x^3y}{2}}=\sqrt{\frac{x}{2y^3}}$

(b) $(a-2b)\cdot\sqrt{\frac{1}{a^2-4b^2}}=\sqrt{(a-2b)^2\cdot\frac{1}{a^2-4b^2}}=\sqrt{\frac{(a-2b)^2}{(a-2b)(a+2b)}}=\sqrt{\frac{a-2b}{a+2b}}$

(c) $(2x-2y)\cdot\sqrt{\frac{1}{8\cdot(x-y)}}=\sqrt{(2x-2y)^2\cdot\frac{1}{8\cdot(x-y)}}=\sqrt{\frac{(2x-2y)\cdot(2x-2y)}{8\cdot(x-y)}}=\sqrt{\frac{2(x-y)\cdot 2(x-y)}{8\cdot(x-y)}}=\sqrt{\frac{x-y}{2}}$

57 (a) $\frac{1}{\sqrt{3}}=\frac{1}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}=\frac{\sqrt{3}}{3}$

(b) $\frac{3}{\sqrt[3]{2}}=\frac{3}{\sqrt[3]{2}}\cdot\frac{\sqrt[3]{2^2}}{\sqrt[3]{2^2}}=\frac{3\cdot\sqrt[3]{4}}{\sqrt[3]{2^3}}=\frac{3\cdot\sqrt[3]{4}}{2}$

(c) $\frac{5}{\sqrt{10}}=\frac{5}{\sqrt{10}}\cdot\frac{\sqrt{10}}{\sqrt{10}}=\frac{5\cdot\sqrt{10}}{10}=\frac{\sqrt{10}}{2}$

(d) $\frac{2}{3\sqrt{6}}=\frac{2}{3\sqrt{6}}\cdot\frac{\sqrt{6}}{\sqrt{6}}=\frac{2\cdot\sqrt{6}}{3\cdot 6}=\frac{\sqrt{6}}{9}$

(e) $\frac{\sqrt{3}}{\sqrt{2}}=\frac{\sqrt{3}}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}=\frac{\sqrt{6}}{2}$

(f) $\frac{12x}{\sqrt[3]{x^2}}=\frac{12x}{\sqrt[3]{x^2}}\cdot\frac{\sqrt[3]{x}}{\sqrt[3]{x}}=\frac{12x\sqrt[3]{x}}{x}=12\sqrt[3]{x}$

58 (a) $\frac{1}{\sqrt{2}+\sqrt{5}}=\frac{1}{\sqrt{2}+\sqrt{5}}\cdot\frac{\sqrt{2}-\sqrt{5}}{\sqrt{2}-\sqrt{5}}=\frac{\sqrt{2}-\sqrt{5}}{2-5}=\frac{\sqrt{2}-\sqrt{5}}{-3}=\frac{\sqrt{5}-\sqrt{2}}{3}$

(b) $\frac{3\sqrt{2}}{\sqrt{6}-\sqrt{5}}=\frac{3\sqrt{2}}{\sqrt{6}-\sqrt{5}}\cdot\frac{\sqrt{6}+\sqrt{5}}{\sqrt{6}+\sqrt{5}}=\frac{3\sqrt{2}\cdot(\sqrt{6}+\sqrt{5})}{6-5}=3\sqrt{12}+3\sqrt{10}=3\sqrt{4\cdot 3}+3\sqrt{10}=6\sqrt{3}+3\sqrt{10}$

(c) $\frac{4\sqrt{3}}{2\sqrt{3}-5\sqrt{2}}=\frac{4\sqrt{3}}{2\sqrt{3}-5\sqrt{2}}\cdot\frac{2\sqrt{3}+5\sqrt{2}}{2\sqrt{3}+5\sqrt{2}}=\frac{4\sqrt{3}\cdot(2\sqrt{3}+5\sqrt{2})}{(2\sqrt{3})^2-(5\sqrt{2})^2}=\frac{8\cdot 3+20\sqrt{6}}{12-50}=\frac{24+20\sqrt{6}}{-38}=-\frac{12+10\sqrt{6}}{19}$

(d) $\frac{\sqrt{2}-\sqrt{5}}{\sqrt{2}+\sqrt{5}}=\frac{\sqrt{2}-\sqrt{5}}{\sqrt{2}+\sqrt{5}}\cdot\frac{\sqrt{2}-\sqrt{5}}{\sqrt{2}-\sqrt{5}}=\frac{(\sqrt{2}-\sqrt{5})^2}{2-5}=\frac{2-2\sqrt{10}+5}{-3}=-\frac{7-2\sqrt{10}}{3}$

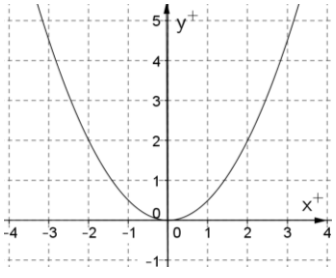
(e) $\frac{x-y}{\sqrt{x}-\sqrt{y}}=\frac{x-y}{\sqrt{x}-\sqrt{y}}\cdot\frac{\sqrt{x}+\sqrt{y}}{\sqrt{x}+\sqrt{y}}=\frac{(x-y)\cdot(\sqrt{x}+\sqrt{y})}{x-y}=\sqrt{x}+\sqrt{y}$

(f) $\frac{9a^2-4b^2}{\sqrt{3a-2b}}=\frac{9a^2-4b^2}{\sqrt{3a-2b}}\cdot\frac{\sqrt{3a-2b}}{\sqrt{3a-2b}}=\frac{(3a-2b)(3a+2b)\sqrt{3a-2b}}{3a-2b}=(3a+2b)\sqrt{3a-2b}$

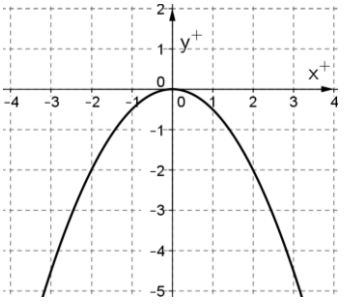
(g) $\frac{a+x}{\sqrt{a^2-x^2}}=\frac{a+x}{\sqrt{a^2-x^2}}\cdot\frac{\sqrt{a^2-x^2}}{\sqrt{a^2-x^2}}=\frac{(a+x)\sqrt{a^2-x^2}}{a^2-x^2}=\frac{(a+x)\sqrt{a^2-x^2}}{(a-x)(a+x)}=\frac{\sqrt{a^2-x^2}}{a-x}$

(h) $\frac{\sqrt{1-x^2}}{\sqrt[3]{1-x^2}}=\frac{\sqrt{1-x^2}}{\sqrt[3]{1-x^2}}\cdot\frac{\sqrt[3]{(1-x^2)^2}}{\sqrt[3]{(1-x^2)^2}}=\frac{\sqrt{1-x^2}\cdot\sqrt[3]{(1-x^2)^2}}{1-x^2}=\frac{\sqrt[6]{(1-x^2)^3}\cdot\sqrt[6]{(1-x^2)^4}}{1-x^2}=\frac{\sqrt[6]{(1-x^2)^7}}{1-x^2}=\frac{(1-x^2)\cdot\sqrt[6]{1-x^2}}{1-x^2}=\sqrt[6]{1-x^2}$

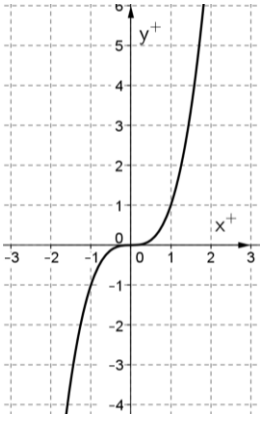
59 (a) $y=0{,}5x^2$



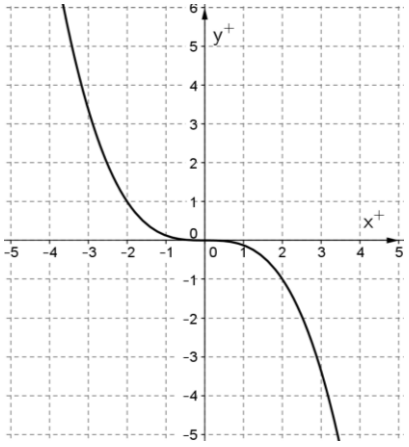
(b) $y=-0{,}5x^2$



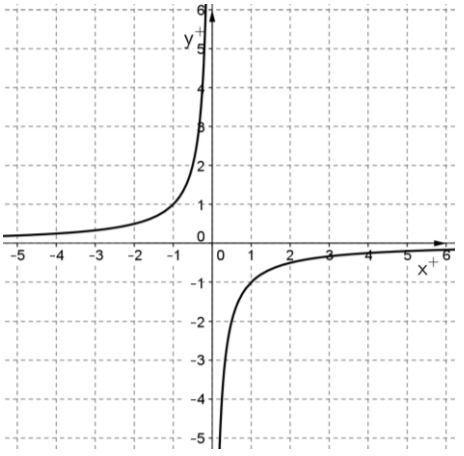
(c) $y=x^3$



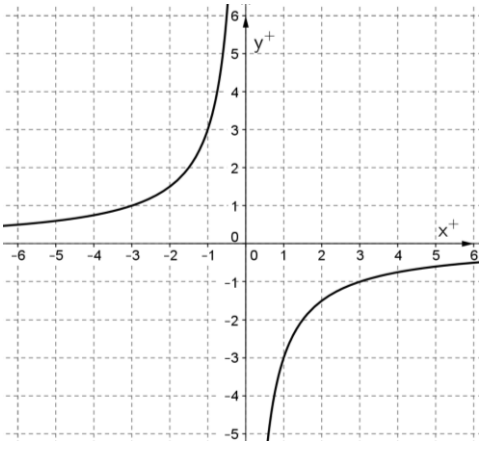
(d) $y = -\frac{1}{8}x^3$



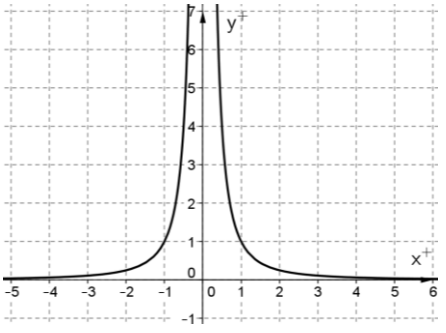
(e) $y = -\frac{1}{x}$



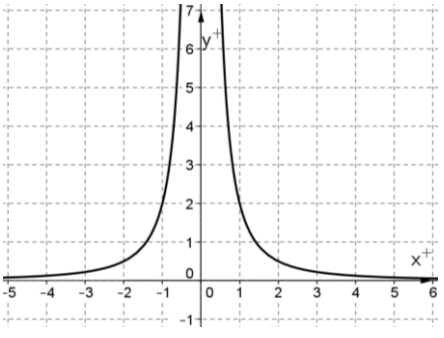
(f) $y = -\frac{3}{x}$



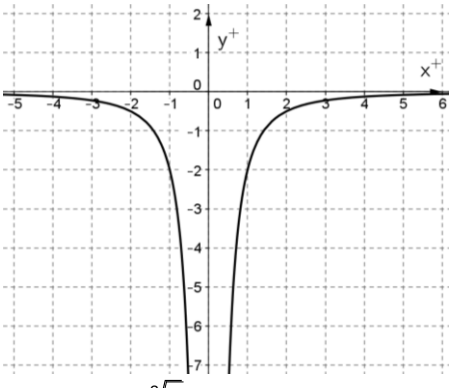
(f) $y = \frac{1}{x^2}$



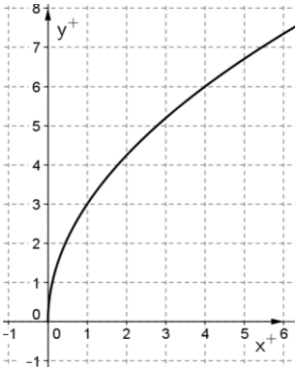
(g) $y = \frac{2}{x^2}$



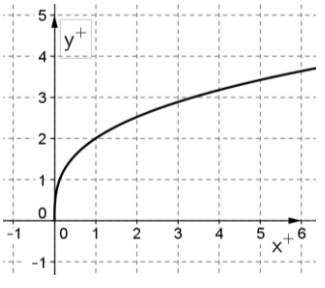
(h) $y = -\frac{2}{x^2}$



60 (a) $y = 3 \cdot \sqrt{x}$



(b) $y = 2 \cdot \sqrt[3]{x}$



61 (a) $5\cdot\sqrt{4x}=16 \quad |^2 \qquad 4x\geq 0$

$25\cdot 4x = 256 \quad |:4 \qquad x\geq 0 \quad \Rightarrow \quad D=\{x\in\mathbb{R}\mid x\geq 0\}$

$25x = 64$

$x=\frac{64}{25} \qquad \frac{64}{25}\in D$

Probe: $5\cdot\sqrt{4\cdot\frac{64}{25}}=5\cdot\sqrt{\frac{256}{25}}=5\cdot\frac{16}{5}=16$ w. A. $\Rightarrow$ $L=\{\frac{64}{25}\}$

(b) $5+\sqrt{x+7}=14 \quad |-5 \qquad x+7\geq 0$

$\sqrt{x+7}=9 \quad |^2 \qquad x\geq -7 \quad \Rightarrow \quad D=\{x\in\mathbb{R}\mid x\geq -7\}$

$x+7=81 \quad |-7$

$x=74 \qquad 74\in D$

Probe: $5+\sqrt{74+7}=5+\sqrt{81}=5+9=16$ w. A. $\Rightarrow$ $L=\{74\}$

(c) $1-2\cdot\sqrt{x}=0 \quad |\ 2\cdot\sqrt{x} \qquad x\geq 0 \quad \Rightarrow \quad D=\{x\in\mathbb{R}\mid x\geq 0\}$

$1=2\cdot\sqrt{x} \quad |^2$

$1=4x$

$x=\frac{1}{4} \qquad \frac{1}{4}\in D$

Probe: $1-2\cdot\sqrt{\frac{1}{4}}=1-2\cdot\frac{1}{2}=1-1=0$ w. A. $\Rightarrow$ $L=\{\frac{1}{4}\}$

(d) $\sqrt{x+27}=9-\sqrt{x} \quad |^2 \qquad x+27\geq 0 \ \wedge \ x\geq 0$

$x+27=81-18\cdot\sqrt{x}+x \qquad x\geq -27 \quad \Rightarrow \quad D=\{x\in\mathbb{R}\mid x\geq 0\}$

$18\cdot\sqrt{x}=54$

$\sqrt{x}=3 \quad |^2$

$x=9 \qquad 9\in D$

Probe: $\sqrt{9+27}=9-\sqrt{9}$

$\sqrt{36}=9-3$

$6=6$ w. A. $\Rightarrow$ $L=\{9\}$

(e) $\sqrt{3x-5}+\sqrt{3x+10}=5 \qquad 3x-5\geq 0 \ \wedge \ 3x+10\geq 0$

$\sqrt{3x-5}=5-\sqrt{3x+10} \quad |^2 \qquad x\geq\frac{5}{3} \qquad x\geq -\frac{10}{3} \qquad D=\{x\in\mathbb{R}\mid x\geq\frac{5}{3}\}$

$3x-5=25-10\cdot\sqrt{3x+10}+3x+10$

$-5=35-10\cdot\sqrt{3x+10}$

$-40=-10\cdot\sqrt{3x+10}$

$4=\sqrt{3x+10} \quad |^2$

$16=3x+10$

$3x=6$

$x=2 \qquad 2\in D$

Probe: $\sqrt{3\cdot 2-5}+\sqrt{3\cdot 2+10}=5$

$\sqrt{1}+\sqrt{16}=5$

$1+4=5$ w. A. $\Rightarrow$ $L=\{2\}$

(f) $\sqrt{2x+11}+\sqrt{2x-5}=8 \qquad 2x+11\geq 0 \ \wedge \ 2x-5\geq 0$

$\sqrt{2x+11}=8-\sqrt{2x-5} \quad |^2 \qquad x\geq -\frac{11}{2} \qquad x\geq\frac{5}{2} \qquad D=\{x\in\mathbb{R}\mid x\geq\frac{5}{2}\}$

$2x+11=64-16\cdot\sqrt{2x-5}+2x-5$

$11=59-16\cdot\sqrt{2x-5}$

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$$11 = 59 - 16 \cdot \sqrt{2x-5}$$
$$-48 = -16 \cdot \sqrt{2x-5}$$
$$3 = \sqrt{2x-5}$$
$$9 = 2x - 5$$
$$2x = 14$$
$$x = 7 \qquad 7 \in D$$

Probe: $\sqrt{2 \cdot 7 + 11} + \sqrt{2 \cdot 7 - 5} = 8$

$$\sqrt{25} + \sqrt{9} = 8$$
$$5 + 3 = 8 \qquad \text{w. A.} \Rightarrow L = \{7\}$$

62 (a) $\sqrt{x+5} + \sqrt{x} = \sqrt{4x+9} \quad |^2 \qquad x + 5 \geq 0 \;\wedge\; x \geq 0 \;\wedge\; 4x + 9 \geq 0$

$$x \geq -5 \;\wedge\; x \geq 0 \;\wedge\; x \geq -\tfrac{9}{4} \qquad D = \{x \in \mathbb{R} \mid x \geq 0\}$$

$$x + 5 + 2 \cdot \sqrt{x+5} \cdot \sqrt{x} + x = 4x + 9$$
$$2 \cdot \sqrt{x \cdot (x+5)} = 2x + 4$$
$$\sqrt{x \cdot (x+5)} = x + 2 \quad |^2$$
$$x \cdot (x+5) = (x+2)^2$$
$$x^2 + 5x = x^2 + 4x + 4$$
$$5x = 4x + 4$$
$$x = 4 \qquad 4 \in D$$

Probe: $\sqrt{4+5} + \sqrt{4} = \sqrt{4 \cdot 4 + 9}$

$$\sqrt{9} + \sqrt{4} = \sqrt{25}$$
$$3 + 2 = 5 \qquad \text{w. A.} \Rightarrow L = \{4\}$$

(b) $4 \cdot \sqrt{x+20} - 3 \cdot \sqrt{x+6} = 7 \cdot \sqrt{x+2} \quad |^2 \qquad x + 20 \geq 0 \;\wedge\; x + 6 \geq 0 \;\wedge\; x + 2 \geq 0$

$$x \geq -20 \;\wedge\; x \geq -6 \;\wedge\; x \geq -2$$
$$D = \{x \in \mathbb{R} \mid x \geq -2\}$$

$$16 \cdot (x+20) - 24 \cdot \sqrt{x+20} \cdot \sqrt{x+6} + 9 \cdot (x+6) = 49 \cdot (x+2)$$
$$16x + 320 - 24 \cdot \sqrt{(x+20) \cdot (x+6)} + 9x + 54 = 49x + 98$$
$$-24 \cdot \sqrt{(x+20) \cdot (x+6)} = 24x - 276 \qquad |:12$$
$$-2 \cdot \sqrt{(x+20) \cdot (x+6)} = 2x - 23 \qquad |^2$$
$$4 \cdot (x^2 + 26x + 120) = 4x^2 - 92x + 529$$
$$4x^2 + 104x + 480 = 4x^2 - 92x + 529$$
$$196x = 49$$
$$x = 0{,}25 \qquad 0{,}25 \in D$$

Probe: $4 \cdot \sqrt{0{,}25 + 20} - 3 \cdot \sqrt{0{,}25 + 6} = 7 \cdot \sqrt{0{,}25 + 2}$

$$4 \cdot \sqrt{20{,}25} - 3 \cdot \sqrt{6{,}25} = 7 \cdot \sqrt{2{,}25}$$
$$4 \cdot 4{,}5 - 3 \cdot 2{,}5 = 7 \cdot 1{,}5$$
$$18 - 7{,}5 = 10{,}5 \qquad \text{w. A.} \Rightarrow L = \{0{,}25\}$$

63 (a) $\sqrt{x+1} + \sqrt{x+8} = \sqrt{x+17} + \sqrt{x-4} \quad |^2 \qquad x + 1 \geq 0 \;\wedge\; x + 8 \geq 0 \;\wedge\; x + 17 \geq 0 \;\wedge\; x - 4 \geq 0$

$$x \geq -1 \;\wedge\; x \geq -8 \;\wedge\; x \geq -17 \;\wedge\; x \geq 4$$
$$D = \{x \in \mathbb{R} \mid x \geq 4\}$$

$$x+1+2\cdot\sqrt{x+1}\cdot\sqrt{x+8}+x+8=x+17+2\cdot\sqrt{x+17}\cdot\sqrt{x-4}+x-4$$
$$9+2\cdot\sqrt{(x+1)\cdot(x+8)}=13+2\cdot\sqrt{(x+17)\cdot(x-4)}$$
$$2\cdot\sqrt{x^2+x+8x+8}=4+2\cdot\sqrt{x^2+17x-4x-68} \qquad |:2$$
$$\sqrt{x^2+9x+8}=2+\sqrt{x^2+13x-68} \qquad |^2$$
$$x^2+9x+8=4+4\cdot\sqrt{x^2+13x-68}+x^2+13x-68$$
$$-4x+72=4\cdot\sqrt{x^2+13x-68} \qquad |:4$$
$$-x+18=\sqrt{x^2+13x-68} \qquad |^2$$
$$x^2-36x+324=x^2+13x-68$$
$$-49x=-392$$
$$x=8 \qquad 8\in D$$

Probe: $\sqrt{8+1}+\sqrt{8+8}=\sqrt{9}+\sqrt{16}$

$3+4=5+2$ w. A. $\Rightarrow$ $L=\{8\}$

(b) $\sqrt{x+13}-\sqrt{x+1}=\sqrt{x+6}-\sqrt{x-2} \quad |^2$

$x+13\geq 0 \;\wedge\; x+1\geq 0 \;\wedge\; x+6\geq 0 \;\wedge\; x-2\geq 0$

$x\geq -13 \;\wedge\; x\geq -1 \;\wedge\; x\geq -6 \;\wedge\; x\geq 2$

$D=\{x\in\mathbb{R}\mid x\geq 2\}$

$$x+13-2\cdot\sqrt{x+13}\cdot\sqrt{x+1}+x+1=x+6-2\cdot\sqrt{x+6}\cdot\sqrt{x-2}+x-2$$
$$14-2\cdot\sqrt{(x+13)\cdot(x+1)}=4-2\cdot\sqrt{(x+6)\cdot(x-2)}$$
$$10-2\cdot\sqrt{x^2+14x+13}=-2\cdot\sqrt{x^2+4x-12} \qquad |:2$$
$$5-\sqrt{x^2+14x+13}=-\sqrt{x^2+4x-12} \qquad |^2$$
$$25-10\cdot\sqrt{x^2+14x+13}+x^2+14x+13=x^2+4x-12$$
$$-10\cdot\sqrt{x^2+14x+13}+38+14x=4x-12$$
$$-10\cdot\sqrt{x^2+14x+13}=-10x-50 \qquad |:(-10)$$
$$\sqrt{x^2+14x+13}=x+5 \qquad |^2$$
$$x^2+14x+13=x^2+10x+25$$
$$4x=12$$
$$x=3 \qquad 3\in D$$

Probe: $\sqrt{3+13}-\sqrt{3+1}=\sqrt{3+6}-\sqrt{3-2}$

$4-2=3-1$ w. A. $\Rightarrow$ $L=\{3\}$

64 (a) $\sqrt{x+5a}-\sqrt{x-a}=\sqrt{4x-10a} \quad |^2$

$x+5a\geq 0 \;\wedge\; x-a\geq 0 \;\wedge\; 4x-10a\geq 0$

$x\geq -5a \;\wedge\; x\geq a \;\wedge\; 4x\geq 10a$

$x\geq \frac{5}{2}a$

$D=\{x\in\mathbb{R}\mid x\geq \frac{5}{2}a\}$ mit $a\in\mathbb{R}^+$

$$x+5a-2\cdot\sqrt{x+5a}\cdot\sqrt{x-a}+x-a=4x-10a$$
$$-2\cdot\sqrt{(x+5a)\cdot(x-a)}=2x-14a \qquad |:2$$
$$-\sqrt{x^2+5ax-ax-5a^2}=x-7a \qquad |^2$$
$$x^2+4ax-5a^2=x^2-14ax+49a^2$$
$$18ax=54a^2 \qquad |:18a \quad \text{mit } a\neq 0 \text{ (Angabe)}$$
$$x=3a \qquad 3a\in D$$

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Probe: $\sqrt{3a+5a}-\sqrt{3a-a}=\sqrt{4\cdot 3a-10a}$

$\sqrt{8a}-\sqrt{2a}=\sqrt{12a-10a}$

$\sqrt{4\cdot 2a}-\sqrt{2a}=\sqrt{2a}$

$2\cdot\sqrt{2a}-\sqrt{2a}=\sqrt{2a}$ w. A. $L=\{3a\}$ mit $a\in\mathbb{R}^+$

(b) $\sqrt{3x+2a}=6+\sqrt{3x-4a}\quad |^2$

$3x+2a\geq 0 \quad\wedge\quad 3x-4a\geq 0$

$3x\geq -2a \quad\wedge\quad 3x\geq 4a$

$x\geq -\frac{2}{3}a \quad\wedge\quad x\geq \frac{4}{3}a$

$D=\{x\in\mathbb{R}\mid x\geq \frac{4}{3}a\}$ mit $a\in\mathbb{R}^+$

$3x+2a=36+12\cdot\sqrt{3x-4a}+3x-4a$

$6a-36=12\cdot\sqrt{3x-4a}\quad |:6$

$a-6=2\cdot\sqrt{3x-4a}$

$a^2-12a+36=4\cdot(3x-4a)$

$a^2-12a+36=12x-16a$

$a^2+4a+36=12x$

$x=\frac{a^2+4a+36}{12}$ $\quad\frac{a^2+4a+36}{12}\in D$, weil $\frac{a^2+4a+36}{12}\geq\frac{4}{3}a \quad |\cdot 12$

$a^2+4a+36\geq 16a$

$a^2-12a+36\geq 0$

$(a-6)^2\geq 0$ w. A.

ein Quadrat ist immer ≥ 0

Probe: $\sqrt{3\cdot\frac{a^2+4a+36}{12}+2a}=6+\sqrt{3\cdot\frac{a^2+4a+36}{12}-4a}$

$\sqrt{\frac{a^2+4a+36}{4}+2a}=6+\sqrt{\frac{a^2+4a+36}{4}-4a}$

$\sqrt{\frac{a^2+4a+36+8a}{4}}=6+\sqrt{\frac{a^2+4a+36-16a}{4}}$

$\sqrt{\frac{a^2+12a+36}{4}}=6+\sqrt{\frac{a^2-12a+36}{4}}$

$\sqrt{\frac{(a+6)^2}{4}}=6+\sqrt{\frac{(a-6)^2}{4}}$

$\frac{a+6}{2}=6+\frac{a-6}{2}$

$\frac{a+6}{2}=\frac{12+a-6}{2}$

$\frac{a+6}{2}=\frac{a+6}{2}$ w. A. $L=\left\{\frac{a^2+4a+36}{12}\right\}$ mit $a\in\mathbb{R}^+$

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W13 S. 28	W14 S. 28	W15 S. 28	W16 S. 29	W17 S. 29	W18 S. 30
W19 S. 31	W20 S. 36	W21 S. 37/39	W22 S. 42	W23 S. 43	W24 S. 43

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